# EXPERIMENTAL VERIFICATION OF A CHUA TYPE MAGNETIZATION MODEL

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Abstract : A Chua type magnetization model which is capable of representing the Rayleigh's relationship, aftereffect and iron loss has been proposed [1,2]. Further, it has been pointed out that a parameter representing hysteretic property of this Chua type model depends on the movement of domain walls and is an important factor related to iron loss of soft magnetic materials [3-5].

In the present paper, the frequency dependence, minor loop, initial magnetization, ferroresonance and transient characteristics of the Chua type model are carefully examined by comparing with the experimental measurements. As a result, it is confirmed that our Chua type model has satisfactory reproducibility of the typical magnetization characteristics commonly observed in practice.

## INTRODUCTION

The models representing magnetization characteristics may be classified into two types. One is a Preisach type model, which assumes that each of domains has a rectangular hysteresis loop, and interaction between domains can be introduced by examining local field acting on domains [6]. Even though the Preisach type model is based on such simple assumptions, it gives valuable results that are in agreement with experimental results [7]. There is an instable problem for which the Preisach's function takes an different value depending on the previous path in the magnetization processes [8]. Further, the Preisach type model has been mathematically generalized to the minor loop, anisotropic and dynamic problems by Mayergoyz [9-11]. The other is a Chua type model, which has been derived on the purely phenomenological behavior of ferromagnetic materials. The key concept of Chua type model is that a trajectory of flux linkage vs. current is uniquely determined by the last point at which the time derivative of flux linkage changes sign [12]. The Chua type model exhibits many important hysteretic properties, e.g. the presence of minor loops and an increase in area of the loop with frequency. Furthermore, it has been reported that a Chua type model [13], and its parameter s can be derived by the Fourier series expansion of the field intersity under the sinusoidal time varying magnetic flux density [14]. Subsequently, a Chua type model has been derived by considering the static and dynamic behaviours of the magnetizing properties. This Chua type model is capable of representing the Rayleigh's relationship, aftereffect and iron loss[1,2]. Also, Sakaki and others have pointed out that a parameter representing the hysteretic property of the Chua type model depends on the movement of domain walls and is an important factor related to the iron loss of soft magnetic materials [3-5].

In this paper, the experimental verifications of the Chua type model are carried out. Namely, the frequency dependence, minor loop, initial magnetization, ferroresonance and transient characteristics of the Chua type model are examined by comparing with the experimental measurements. As a result, it is confirmed that the Chua type model has satisfactory reproducibility of the typical magnetization characteristics.

## THE CHUA TYPE MAGNETIZATION MODEL

### Derivation of model

Since the solutions of magnetization model exhibit various magnetization properties such as the hysteresis, saturation and minor loops, then the model itself must be composed of the parameters not affected by the past magnetizing hystories. One of the characteristics not affected by the past hystories is an ideal or anhysteretic magnetization curve, which can be obtained by first applying the superposed static and alternating fields, and then reducing the alternating field to zero and observing the flux density. This ideal magnetization curve may be represented by

$$H = (1/\mu)B, \tag{1}$$

where H, B and $\mu$ are the field intensity, flux density and a parameter defined by $\mu = B/H$, respectively.

The other characteristic not affected by the past hystories is the reversible permeability defined by

$$\mu_r = \Delta B/\Delta H, \tag{2}$$

where $\Delta B$ and $\Delta H$ are respectively the infinitesimally small alternating flux density and field intensity accompanying with the measurement process of anhysteretic magnetization curve. Fig. 1 shows a relationship between the anhysteretic magnetization curve and associated reversible permeability. From an experimental point of view, the magnetization is accomplished in essence through the time variations of flux density dB/dt and field intensity dH/dt. Thereby, (2) may be rewritten by

$$dH/dt = (1/\mu_r)dB/dt. \tag{3}$$

After introducing a hysteresis coefficient s [ohm/m] into (3), consideration of total field intensity due to the static characteristic (1) and dynamic characteristic (3) yields a following relation :

$$H + (\mu_r/s)dH/dt = (1/\mu)B + (1/s)dB/dt, \tag{4}$$

where $\mu$, $\mu_r$ and s can be represented by the following functional forms :

$$\left.\begin{aligned} \mu &= f_\mu(B), \\ \mu_r &= f_r(B), \\ s &= f_s(B, dB/dt, dH/dt). \end{aligned}\right\} \tag{5}$$

Equation (4) is the Chua type model. The parameters $\mu$, $\mu_r$ and s in (4) for a soft iron are respectively given in Figs. 2(a), 2(b) and 2(c).

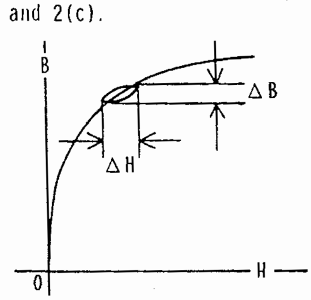


Fig. 1. Anhysteretic magnetization curve and reversible permeability $\mu_r = \Delta B/\Delta H$.

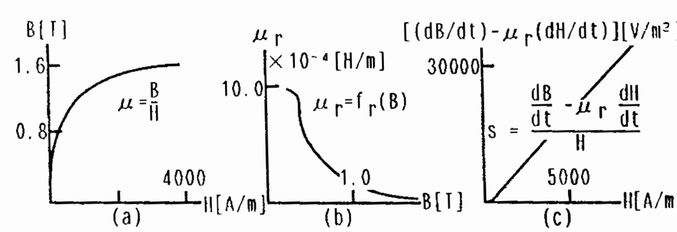


Fig. 2. (a) Anhysteretic magnetization $\mu = f_\mu(B)$, (b) reversible permeability $\mu_r = f_r(B)$ and (c) hysteresis coefficient $s = f_s(B, dB/dt, dH/dt)$ characteristics of a soft-iron.

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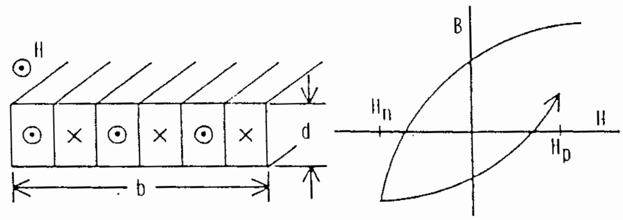



(a) (b)
Fig. 3. (a) Barlike domain walls model.
(b) Definition of $H_p$ and $H_n$.

Parameter s

We have introduced the parameter s to derive the Chua type model (4). When we assume a barlike domain walls model shown in Fig. 3(a), then it is possible to derive a following relation between the parameter s and number of domain walls n:

$$s = \pi^3 \rho n/4.2bd \ , \qquad (6)$$

where $\rho$, b and d are respectively the resistivity, thickness and width of material [5]. According to the Sakaki and others intensive works , consideration of the anomalous eddy currents has lead that the number of barlike domain walls n in (6) is proportional to the flux density B as well as frequency f [3-5]. This means that the parameter s in (4) and (6) must be represented as a function of the flux density B as well as frequency f. It is obvious that the parameter s in Fig. 2(c) has been defined in accordance with this fact.

When we define the reversing and applying point field intensities $H_n$, $H_p$ as shown in Fig. 3(b), then a Preisach's function $\psi$ can be represented by [7]

$$\psi = \partial^2 B/\partial H_p \partial H_n \ . \qquad (7)$$

According to the Refs. 1 and 2, the Preisach function $\psi$ in (7) is related with the parameter s in (4) and (6) by

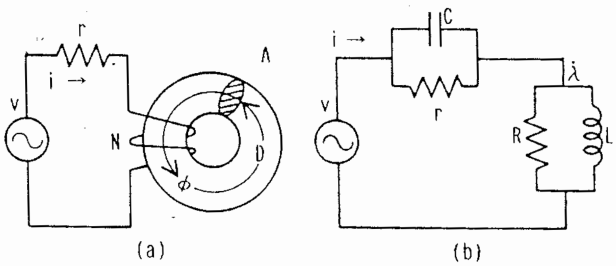
$$\psi = s/(\partial H/\partial t) \ , \qquad (8)$$

$$\propto n/(\partial H/\partial t) \ . \qquad (9)$$

Since the number of domain walls n in (9) is proportional to the frequency f and the time derivative of field intensity $\partial H/\partial t$ is approximately proportional to the frequency f , the Preisach's function $\psi$ may be assumed independent to the frequency f. Namely, it may be considered that the Preisach's function $\psi$ represents the flux density dependency of the number of domain walls n. Further, the number of domain walls n is proportional to the flux density B, and this flux density B is approximately proportional to the field intensity H in the weakly magnetized region so that the Preisach's function $\psi$ in the weakly magnetized region reduces to a constant. This weakly magnetized region and the constant may be called the Rayleigh region and the Rayleigh constant, respectively [15].

Lumped circuit model

In order to derive a lumped circuit model of the Chua type model, consider a simple reactor shown in Fig.4(a), it is possible to write a following equation from (4) :




(a) (b)
Fig. 4. (a) A simple troidal core, and
(b) its electrical equivalent circuit.

$$\int_0^D [H+(\mu_r/s)dH/dt]dl = \int_0^D [(1/\mu)B+(1/s)dB/dt]dl \ , \qquad (10)$$

where D and dl denote the mean length of flux path and infinitesimally small distance along the flux path D , respectively. With A denoting a cross-sectional area normal to the flux path, the right hand term of (10) can be rewritten by

$$\int_0^D [(1/\mu)B+(1/s)dB/dt]dl = (1/L_i)\phi+(1/R_i)d\phi/dt \ , \qquad (11)$$

where $\phi = AB$, $L_i = \mu A/D$ and $R_i = sA/D$. Also, the left hand term of (10) can be represented in terms of the current i and the number of turns N as

$$\int_0^D [H+(\mu_r/s)dH/dt]dl = Ni+N(\mu_r/s)(di/dt) \ . \qquad (12)$$

The relationship among the flux $\phi$, current i, voltage v and electrical resistance r of coil is given by

$$i = (1/r)[v-N(d\phi/dt)] \ , \qquad (13)$$

or

$$di/dt = (1/r)[(dv/dt)-N(d^2\phi/dt^2)] \ . \qquad (14)$$

By means of (11)-(14) , it is possible to derive a following lumped circuit equation :

$$\frac{N}{r}\left[v+\frac{\mu_r}{s}\frac{dv}{dt}\right] = \frac{N^2}{r}\frac{\mu_r}{s}\frac{d^2\phi}{dt^2} + \left[\frac{N^2}{r}+\frac{1}{R_i}\right]\frac{d\phi}{dt} + \frac{1}{L_i}\phi \ , \qquad (15)$$

or

$$\frac{1}{r}\left[v-\frac{d\lambda}{dt}\right] + C\frac{d}{dt}\left[v-\frac{d\lambda}{dt}\right] = \frac{\lambda}{L} + \frac{1}{R}\frac{d\lambda}{dt} \ , \qquad (16)$$

where $\lambda = N\phi$, $L = N^2 L_i$, $R = N^2 R_i$ and $C = (\mu_r/s)(1/r)$.

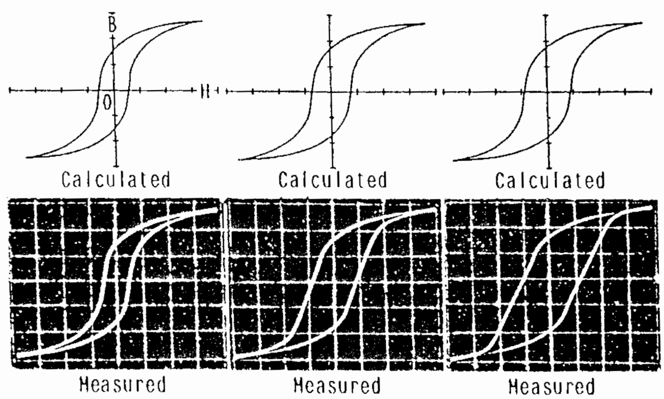
By considering (16), it is possible to draw an electrical equivalent circuit of a toroidal reactor shown in Fig. 4(b).

Experimental verification

Equation (16) was solved by means of the conventional trapezoidal rule under various conditions. Various constants used in the calculations are listed in table 1. The

Table 1. Various constants used in the calculations.

| | |
|---|---|
| Area normal to the flux path | A = 0.0001 [$m^2$] |
| Mean length of magnetic flux path | D = 0.28 [m] |
| Number of turns of coil | N = 900 [Turn] |
| Electric resistance of coil | r = 6.3 [$\Omega$] |
| Capacitance of R-L-C circuit | c = 100 [$\mu$F] |




(a) f=10[Hz] (b) f=70[Hz] (c) f=120[Hz]
Fig. 5. Steady state hysteresis loops under the different exciting frequencies.
H:636[A/m]/div., B:0.552[T]/div.

parameters $\mu$, $\mu_r$ and s of the Chua type model (4) were obtained from the Fig. 2(a), 2(b) and 2(c), respectively.

At first , we examined the frequency characteristic of the model. Figure 5 shows the steady state hysteresis loops obtained under the different exciting frequencies together with the experimental measurements.

Second, we examined an initial magnetization process. Its result is shown in Fig. 6.

Third , in order to examine the minor loop and aftereffect properties , we calculated a magnetization process observed in a half wave rectifier circuit. Fig. 7 shows the magnetization process in a half wave rectifier circuit.

Finally, we calculated a transient current of a R-L-C series resonance circuit. As shown in Fig. 8, ferroresonance phenomena is obviously observed.

Comparison the calculated with measured results in Fig. 5-8 reveals that the Chua type model has satisfactory reproducibility of the typical magnetization process.

## CONCLUSION

As shown above, we have clarified that our Chua type model is capable of representing the typical magnetization characteristics, such as the frequency dependency, minor loop, aftereffect and ferroresonance.

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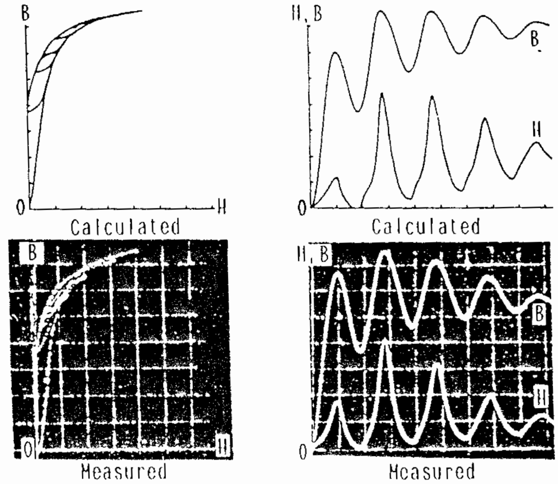


Fig. 6. (a) An initial magnetization curves, (b) Wave forms of the flux density B and Field intensity H. t:10.0[msec]/div., H:1273[A/m]/div., B:0.22[T]/div.

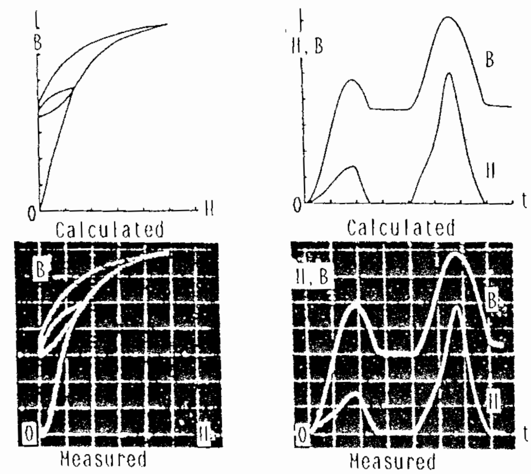


Fig. 7. (a) Hysteresis loops in a half wave rectifier circuit , and (b) waveforms of the flux density B and field intensity H. t:5.0[msec]/div., H:636[A/m]/div., B:0.22[T]/div.

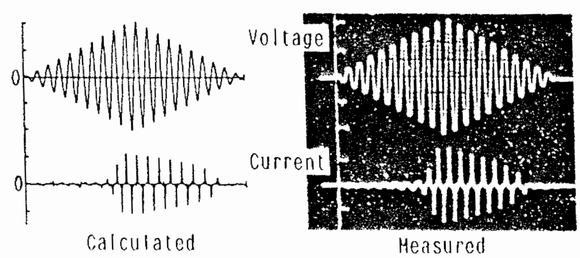


Fig. 8. Transient current i and imposed voltage v in R-L-C ferroresonanct circuit. t:50[msec]/div., Voltage:20[V]/div., Current:5[A]/div.