AN EFFICIENT COMPUTATION OF SATURABLE MAGNETIC FIELD PROBLEM USING LOCALLY ORTHOGONAL DISCRETIZATION

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Abstract:Previously,faster magnetic field computation using locally orthogonal discretization, was proposed for two-dimensional magnetic field problems [1-3]. This method is now applied to the saturable magnetic field problems. The locally orthogonal discretization method requires a single potential to establish the upper and lower bounded solutions so that the functional as well as potential are evaluated in the most efficient manner. Application of this new method to the saturable magnetic field problems yields the excellent results even though a small number of elements is employed.

INTRODUCTION

Recently, the finite element method has been extensively applied to the various engineering and physical problems. When we apply the finite element method to the field problems, then it is difficult to know how close the numerical solution is correct solution. To overcome this difficulty, a dual energy finite element method which yields upper and lower bounded solutions to the correct solution has been proposed [4,5]. Nevertheless, it remains a serious problem that the dual energy finite element method yields an improved functional but never provide the improved local solutions with lower computational cost. To remove this difficulty, a new method which uses a single potential based on the geometrical dual property of locally orthogonal discretization has been proposed[1-3].

In this paper, we apply this new method to the saturable magnetic field problem. As a result,fairly reduction of the computational cost and time can be achieved by the locally orthogonal discretization method for the saturable magnetic field problems.

THE LOCALLY ORTHOGONAL DISCRETIZATION METHOD

Basic equations

On the two-dimensional x-y plane, magnetic field H is related with the current density J as

$$\int_c H \cdot dl = \int_s J \cdot ds \quad , \tag{1}$$

where c is the path enclosing the area s. Magnetic flux density B is given by

$$B = \mu H \quad , \tag{2}$$

where μ is a permeability of medium. The flux density B has to satisfy the following condition

$$\nabla \cdot B = 0 \quad . \tag{3}$$

In order to satisfy the condition (3), it is assumed that the flux density B is represented by a vector potential A , viz.,

$$B = \nabla \times A \quad . \tag{4}$$

By means of Eqs.(2) and (4), a governing equation of two-dimensional magnetostatic field is written by

$$\int_c (1/\mu) \nabla \times A \cdot dl - \int_s J \cdot ds = 0 \quad . \tag{5}$$

Voronoi-Delaunay diagram

Dalaunay triangulation of arbitrary set of points is constructed by considering the properties of its geometric dual i.e., the set of Voronoi polygons. Delaunay triangles are related to Voronoi polygons in that the circumcenters of Delaunay triangles are the vertices of the Voronoi polygons. Figure 1 shows the triangles in a Delaunay mesh. The Voronoi polygons

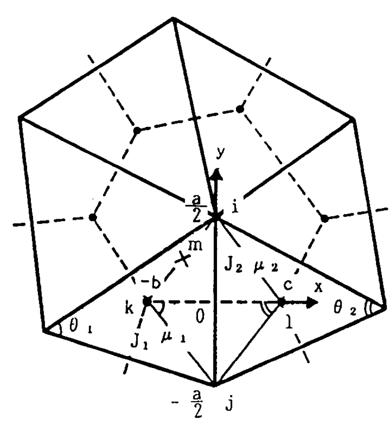

Fig. 1. Voronoi-Delaunay diagram , and locally orthogonal coordinate.

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associated with these Delaunay triangles are shown by dashed lines in this figure. By considering Fig.1, it is obvious that the Delaunay triangles and the Voronoi polygons are locally orthogonal:each triangle side is perpendicular to the corresponding Voronoi polygon edge. Further, two complete but independent sets of nodal variables may be defined on this Voronoi-Delaunay diagram : one is located at the vertices of the Delaunay triangles ; and the other is located at the vertices of the Voronoi polygons. Thus, the govening equation (5) may be discretized by the Delaunay mesh or the Voronoi mesh system.

Node equations

On the local coordinate system shown in Fig.1 , nodes i and j are located on the boundary between the regions 1 and 2, where it is assumed that each of the Delaunay triangles takes a distinct permeability μ. This means that a flux desity B_x ($=\partial A/\partial y$) in the direction of x-axis is common to both regions 1 and 2 in Fig.1. A simple Lagrange interpolation between the nodes i and j in Fig.1 yields a trial function for the Delaunay system as

$$A_p = (1/2)(A_i + A_j) + (A_i - A_j)(y/a) \quad , \tag{6}$$

where a is the distance between nodes i and j. Integrating over a portion of the Voronoi polygon enclosing node i after substituting Eq.(6) into Eq.(5) yields

$$\begin{aligned} &\int_{-b}^{c}(1/\mu)\,\partial A_p/\partial y\; dx - \int_{-b}^{c}\int_{0}^{a/2} J\; dxdy \\ &= (1/2)\{(1/\mu_1)\cot\theta_1 + (1/\mu_2)\cot\theta_2\}(A_i - A_j) \\ &\quad - (1/4)(abJ_1 + acJ_2) = 0 \quad , \end{aligned} \tag{7}$$

Integration of Eq.(5) for the other portions of Voronoi polygon enclosing node i is carried out in the same way as Eq.(7). The full set of node equations gives a Delaunay triangle system of equations, which satisfies the continuity of flux density between the adjacent Delaunay triangles.

On the other hand, the nodes k and l are located on the x-axis in Fig.1. In this case, a field intensity H_y [$= -(1/\mu)\,\partial A/\partial x$] is common to both regions 1 and 2. In order to satisfy this boundary condition, it is essential to employ the two different trial functions given by

$$\begin{aligned} A_c = \{&(A_k/\mu_1 b) + (A_l/\mu_2 c) + (A_l - A_k)x/(\mu_2 bc)\} \\ &/\{(1/\mu_1 b) + (1/\mu_2 c)\} \quad , -b \leqq x \leqq 0 \quad , \end{aligned} \tag{8a}$$

$$\begin{aligned} A_c = \{&(A_k/\mu_1 b) + (A_l/\mu_2 c) + (A_l - A_k)x/(\mu_1 bc)\} \\ &/\{(1/\mu_1 b) + (1/\mu_2 c)\} \quad , 0 \leqq x \leqq c \quad . \end{aligned} \tag{8b}$$

Integrating over a portion of the Delaunay triangle enclosing node k after substituting Eqs.(8a) and (8b) into Eq.(5) yields

$$\begin{aligned} &\int_{-a/2}^{a/2}(1/\mu)(-\partial A_c/\partial x)\; dy - \int_{-a/2}^{a/2}\int_{-b}^{0} J\; dxdy \\ &= \{2/(\mu_1\cot\theta_1 + \mu_2\cot\theta_2)\}(A_k - A_l) \\ &\quad - (1/2)abJ_1 = 0 \; . \end{aligned} \tag{9}$$

Integration of Eq.(5) for the other portions of Delaunay triangle enclosing node k is carried out in the same way as Eq.(9). The full of node equations gives a Voronoi system of equations, which satisfies the common field intensity between the adjacent Delaunay triangles.

Determination of permeability μ

On the saturable magnetic field, the permeability μ in Eq.(5) is essentially represented as a function of the flux density B or field intensity H. In this paper, it has been assumed that the permeability μ takes a distinct value in each of the Delaunay triangles, so that a magnitude of the flux density or field intensity in a Delaunay triangle can be easily calculated by means of the two orthogonal components of the flux density or field intensity. The Delaunay system of equations has been derived by using the continuity of flux density between the Delaunay triangles. This means that the permeability μ of Delaunay system must be determined as a function of the flux density B. On the other side, the Voronoi system of equations has been derived by using the common field intensity between the Delaunay triangles. This means that the permeability μ of Voronoi system must be determined as a function of the field intensity H.

Hybrid potential

At the interface between the different materials taking the different permeabilities, both of the continuity of flux density and the common field intensity conditions must be simultaneously satisfied. In the most of the numerical methods,one of these boundary conditions is rigorously satisfied and the other is approximately satisfied. The Delaunay system rigorously satisfies the condition of flux density. On the other side, the Voronoi system rigorously satisfies the common field intensity condition. Therefore, both of the Delaunay and the Voronoi systems complement the boundary conditions each other. This suggests that the convergence of the numerical solution may be accelerated by averaging the approximate solutions in Eq.(7) and Eq.(9). For example,the midside potential A_m in Fig.1 is given by

$$A_m = (1/2)(A_i + A_k) \quad . \tag{10}$$

An example

To illustrate the method, we applied it to the calculation of the magnetic field in ferromagnetic material of square cross-section. By symmetry, only 1/4 part of the square must be computed as shown in Fig.2. Figure 3 shows a B-H characteristic of the ferromagnetic material.

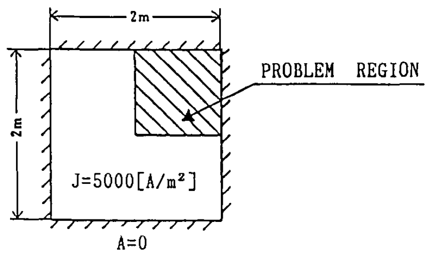

Fig. 2. A ferromagnetic conductor of square cross-section.

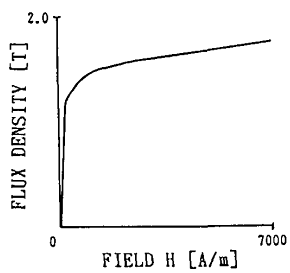

Fig. 3. B-H chracteristic of the ferromagnetic conductor.

Figure 4 shows the results of computations obtained by the Delaunay and Voronoi systems. At the points of inflection on the equipotential lines in Fig.4, notable difference between the Delaunay and Voronoi solutions may be observed. Figure 5(a) shows a hybrid potential distribution obtained from the results in Fig.4. By considering the results in Fig.5(a), it is obvious that our new method provides an excellent result compared with the traditional first order finite element solution in Fig.5(b). It must be noted that each of the Fig.4(a) and Fig.4(b) has only 1/4 of the nodes employed in Fig.5(b).

CONCLUSION

As shown above, our locally orthogonal discretization method based on the geometrical duality of Delaunay triangles and Voronoi polygons is still effectively applied to the saturable magnetic field problems. To obtain solutions of similar accuracy in an example problem, the new method required about 1/4 of the nodes and considerably less computer time than the traditional first order finite element method.

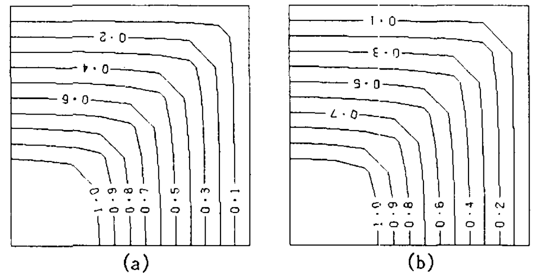

Fig. 4. Computational results. (a)Delaunay system solution ; (b)Voronoi system solution ;and both systems evaluated the 64 node potentials.

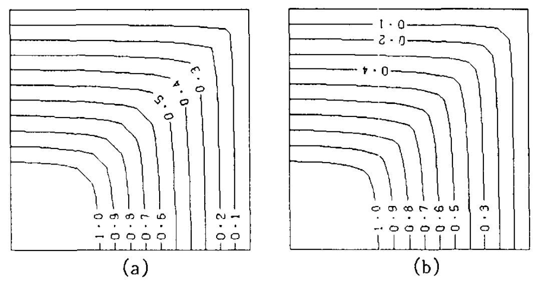

Fig. 5. Comparison of the traditional first order finite element with the new method. (a)Hybrid potential obtained from the Delaunay and Voronoi solutions in Fig.4. (b)The traditional first order finite element solutions, where the 256 node potentials were evaluated.

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