APPLICATION OF A CHUA TYPE MODEL TO THE LOSS AND SKIN EFFECT CALCULATIONS

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Abstract: Previously,we have proposed a specific Chua type model, and shown that our model is closely related with the Preisach and Rayleigh models. In the present paper, a simple linearized Chua type model is proposed for the loss and skin effect calculations. As a result, it is found that the hysteresis makes the skin depth deeper.

INTRODUCTION

Recently,considerable effort has been done to represent the magnetization characteristics for the computer oriented design of magnetic devices [1-3]. The models representing magnetization characteristics may be classified into two types. One is a Preisach type model, which assumed that each of domains has a rectangular hysteresis loop and interaction between domains can be introduced by assuming local field acting on domains [4]. Even though the Presisach type model is based on such simple assumptions, it gives valuable results that are in agreement with experimental results [5]. There is an instable problem for which the Preisach function takes an different value depending on the previous path in the magnetization processes [6]. The other is a Chua type model,which is based on the fact that a trajectory of flux linkage vs. current is uniquely determined by the last point at which the time derivative of flux linkage changes sign. The Chua type model exhibits many important hysteretic properties, e.g. the presence of minor loops and an increase in area of the loop with frequency. Moreover, the Chua type model has been successfully applied to the three-dimensional magnetic field problems and also to the power electronics circuits [7-10].

In the present paper, we show that linearization of a specific Chua type model leads to an elliptical approximation of hysteresis loop. By means of this linearized Chua type model,the hysteresis loss formulas are derived. Also, it is shown that an application of this linearized model to a simple one dimensional magnetic field problem gives an interesting relationship between the skin depth and hysteretic property.

THE MAGNETIZATION MODEL

Chua Type Model

The magnetization characteristics are divided into the two-major properties. One is a saturation property which is usually represented in terms of a permeability μ. The other is a hysteretic property which causes a time lagging flux density B behind the field intensity H. Chua type model assumes that these two-properties are combined into

$$H = (1/\mu)B+(1/s)dB/dt, \tag{1}$$

where s,t are respectively the hysteresis coefficient and time [1,2]. In (1), the term $(1/\mu)B$ represents the saturation property and the other term $(1/s)dB/dt$ represents the hysteretic property. The parameters μ and s in (1) are determined by considering the conditions dB/dt=0 and B=0,respectively [11].

Also, a recent paper has shown that the hystersis coefficient s in (1) is related with the Preisach function Ψ [1] as

$$s = \Psi(\partial H/\partial t). \tag{2}$$

A Specific Chua Type Model

By considering the time derivative term dB/dt in (1), it is revealed that a term $\mu_r(dH/dt)$ (where μ_r is a reversible permeability) must be included in dB/dt, so that (1) may be modified to

$$H = (1/\mu)B+(1/s)[(dB/dt)-\mu_r(dH/dt)]. \tag{3}$$

(3) is a specific Chua type model,and it is possible to show that a relationship between the hysteresis coefficient s in (3) and Preisach function Ψ is still held as (2). An alternative derivation of (3) has been reported in [2]. By means of (2), it is possible to write (3) as

$$H+(\mu_r/\Psi) = (1/\mu)B+(1/\Psi)\partial B/\partial H. \tag{4}$$

When we assume that the parameters μ,μ_r,Ψ in (4) are constant values,then an initial magnetization curve by (4) can be obtained as

$$B = \mu H+(\mu/\Psi)(\mu_r-\mu)[1-\exp(-\Psi H/\mu)]$$
$$\sim \mu_r H+(1/2)\Psi H^2, \tag{5}$$

where $\mu_r \ll \mu$, $\exp(-\Psi H/\mu) \sim 1-(\Psi H/\mu)+(1/2)(\Psi H/\mu)^2$ are assumed.

In the weakly magnetized region,following Rayleigh's initial magnetization curve is established:

$$B = \mu_i H+(1/2)\nu H^2, \tag{6}$$

where μ_i,ν are respectively the initial permeability and Rayleigh constant [12]. Comparison (5) with (6) reveals that the specific Chua type model (3) gives the Rayleigh's initial magnetization curve when the intial permeability μ_i and Rayleigh constant ν are respectively equivalent to the reversible permeability μ_r and Preisach function Ψ.

Hysteresis Loss

Let the parameters μ,μ_r,s in (3) be set by the constant values,then (3) becomes to a linear magnetization model whose hysteresis loops are approximated by the elliptical loops. By means of this linearized model, the hysteresis loss is given by

$$P_h = (1/2)[\mu(\mu-\mu_r)/(s^2+\omega^2\mu^2)]s\omega^2 H_m^2, \tag{7}$$

where a sinusoidal steady state has been assumed; ω,H_m are respectively the angular velocity and maximum field intensity.

In the weakly magnetized region, the hysteresis coefficient s is expressed by

$$s = \Psi\omega H_m, \tag{8}$$

because (2) is held. Moreover, by means of (5) and (6), the Preisach function Ψ and reversible permeability μ_r are respectively equivalent to the Rayleigh constant ν and intial permeability μ_i so that

$$\Psi H_m = \nu H_m = (B_m/H_m)-\mu_i = \mu-\mu_i = \mu-\mu_r. \quad (9)$$

Substitution of (8) and (9) into (7) yields

$$P_h = [\mu(\mu-\mu_r)\Psi\pi f H_m^3]/[(\mu-\mu_r)^2+\mu^2] \sim (\pi/2)f\Psi H_m^3, \quad (10)$$

where $\omega=2\pi f$ (f:frequency); and $\mu_r \ll \mu$ is assumed. (10) is a hysteresis loss formula in the weakly magnetized region. On the other side, the hysteresis loss by Rayleigh loop is

$$P_h = (4/3)f\Psi H_m^3, \quad (11)$$

where $\nu=\Psi$ is assumed [12]. Comparison (10) with (11) reveals that (10) has essentially similar nature to those of (11) but a constant term $\pi/2$ in (10) is somewhat larger than (4/3) in (11). Thus, our elliptical approximation model gives a loss formula (10) corresponding to Rayleigh model in the weakly magnetized region.

In the highly magnetized region, the hysteresis coefficient s is approximately given by

$$s = \omega(B_m-\mu_r H_m)/H_c \sim \omega B_m/H_c, \quad (12)$$

where B_m, H_c are respectively the maximum flux density and coercive field; and $B_m \gg \mu_r H_m$ is assumed. Substitution of (12) into (7) yields

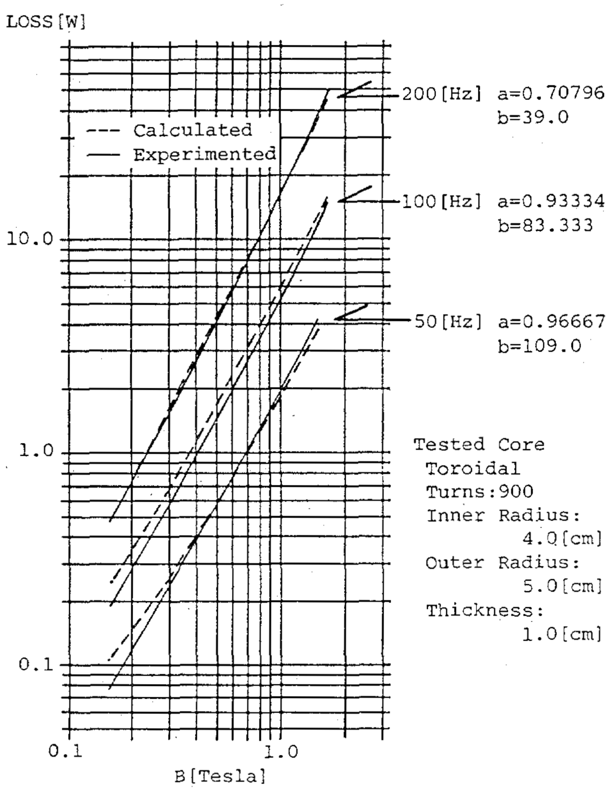

Fig. 1. Examples of hysteresis loss calculation.

$$P_h = (1/2)\mu\omega B_m H_c H_m^2(\mu-\mu_r)/[B_m^2+(\mu H_c)^2] \sim \pi f B_m H_c = \pi f B_m(a+bB_m), \quad (13)$$

where $B_m \sim \mu H_m$, $H_m \gg H_c$ are assumed. The coercive force H_c has been approximated by $H_c=a+bB_m$ in (13), and its parameters a,b must be experimentally determined. (13) is a hysteresis loss formula in the highly magnetized region, and has a simple physical meaning that the term $B_m H_c$ represents an area enclosed by hysteresis loop. Some of the results by (13) are shown in Fig. 1.

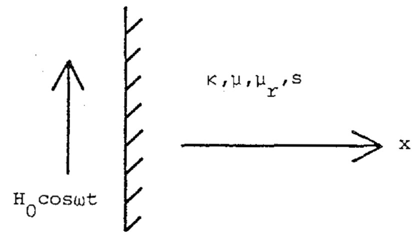

Fig. 2(a). One-dimensional magnetic field problem,where x=0 $H=H_0\cos(\omega t)$ and $x=\infty$ H=0.

Skin Effect

In order to show the effect of the skin depth in hysteretic property, the linearized Chua type model is applied to a simple one-dimensional magnetic field problem shown in Fig. 2(a). The governing equations in the low frequency range are

$$\nabla\times E = -\partial B/\partial t, \quad \nabla\times H = J, \quad J = \kappa E, \quad (14)$$

where E,J,κ are respectively the electric field intensity, current density and conductivity. By means of (3) and (14), it is possible to derive a following equation

$$\nabla^2 B+(\mu/s)\nabla^2(\partial B/\partial t) = \kappa\mu(\partial B/\partial t)+(\kappa\mu\,\mu_r/s)(\partial^2 B/\partial t^2). \quad (15)$$

Application of (15) to the problem region in Fig. 2(a) yields

$$B = \mu H_0 Z\exp(-\sqrt{W}x)\cos(\omega t-\sqrt{Y}x-\delta), \quad (16)$$

where

$$Z = \sqrt{[s^2+(\omega\mu_r)^2]/[s^2+(\omega\mu)^2]},$$

$$W = \omega\kappa\mu\{\omega(\mu-\mu_r)s+\sqrt{[\omega(\mu-\mu_r)s]^2+[s^2+\omega^2\mu\mu_r]^2}\}/[2(s^2+\omega^2\mu^2)]$$

$$Y = \omega\kappa\mu\{-\omega(\mu-\mu_r)s+\sqrt{[\omega(\mu-\mu_r)s]^2+[s^2+\omega^2\mu\mu_r]^2}\}/[2(s^2+\omega^2\mu^2)]$$

$$\delta = \tan^{-1}(\omega\mu/s) - \tan^{-1}(\omega\mu_r/s). \quad (17)$$

Furthermore, by means of (3),(16) and (17), the magnetic field intensity H in the magnetic material can be obtained as

$$H = H_0\exp(-\sqrt{W}x)\cos(\omega t-\sqrt{Y}x). \quad (18)$$

The hysteresis loops in the magnetic material can be obtained from (16) and (18). Fig. 2(b) shows the examples of hysteresis loop in the magnetic material.

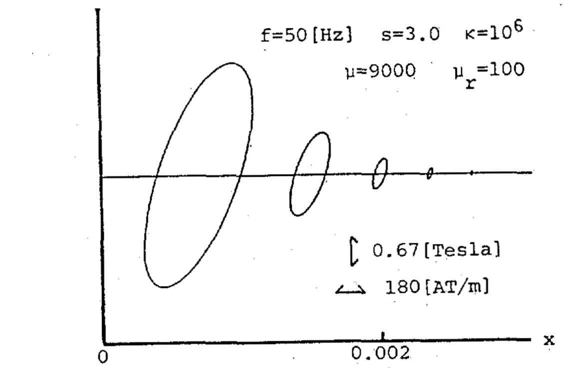

Fig. 2(b). Examples of hysteresis loops in the magnetic materials.

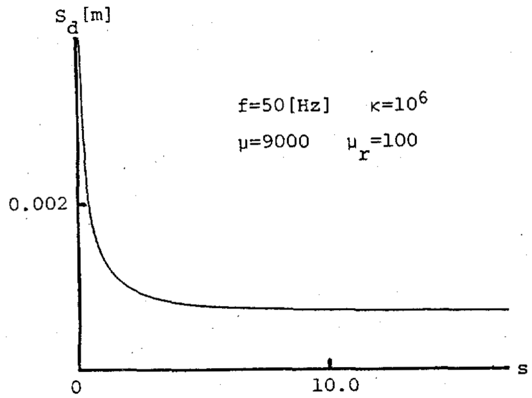

Fig. 2(c). The effect of the skin depth in hysteretic property.

The skin depth s_d is given from (16) as

$$s_d = 1/\sqrt{W}. \tag{19}$$

Fig. 2(c) shows the effect of the skin depth in hysteretic property. Let consider the two-ultimate conditions: one is $s \to \infty$ (no hysteresis) and the other is $s \to 0$ (infinitely hysteresis), then the skin depth s_d becomes to

$$\left.\begin{aligned} s \to \infty, \ s_d &= \sqrt{2/\omega\kappa\mu}, \\ s \to 0, \ s_d &= \sqrt{2/\omega\kappa\mu_r}. \end{aligned}\right| \tag{20}$$

Since, in common case, the permeability μ is greater than the reversible permeability μ_r, then (20) suggests that the hysteresis makes the skin depth deeper.

CONCLUSION

As shown above, we have elucidated that a specific Chua type model is capable of representing the hysteresis loss, and that the skin depth may be greatly dominated by the hysteretic property.

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