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DIGITAL SIMULATION OF POLYPHASE INDUCTION MOTORS

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The fundamental equations of polyphase induction motors (linear simultaneous differential equations with periodic coefficients) are directly solved by finite difference methods for balanced and for unbalanced conditions.

Notation

A	= parameter in approximate exponential function
$\boldsymbol{C}$	= current connection matrix
$\boldsymbol{G}[t, \omega_m]$	= $(1/\omega_m)(\mathrm{d}/\mathrm{d}t)\boldsymbol{L}[t, \omega_m]$, torque matrix
$\boldsymbol{I}[t]$	= $\{i_a, i_b, i_c, i_d, i_e, i_f\}$, current vector
$\boldsymbol{I}'[t]$	= $\{i_1, i_2, i_3, i_4\}$, new coordinate current vector
$\boldsymbol{L}[t, \omega_m]$	= $\boldsymbol{l} + \boldsymbol{L}' + \boldsymbol{M}[t, \omega_m]$, inductance matrix
$L_{ij} \cos\left(\frac{i-j}{3} 2\pi\right)$	= element in i-th row and j-th column of self-inductance matrix $\boldsymbol{L}'$
$\boldsymbol{L}'$	= self-inductance matrix
$\boldsymbol{l}$	= $[l_a, l_b, l_c, l_d, l_e, l_f]$, leakage inductance matrix (diagonal)
$\boldsymbol{M}[t, \omega_m]$	= mutual inductance matrix
$M_{ij} \cos\left(\omega_m t + \frac{i-j}{3} 2\pi\right)$	= element in i-th row and j-th column of mutual inductance matrix $\boldsymbol{M}[t, \omega_m]$
p	= number of pole pairs
$\boldsymbol{R}$	= $\lceil r_a, r_b, r_c, r_d, r_e, r_f \rfloor$, resistance matrix (diagonal)
$\boldsymbol{S}[t, \omega_m]$	= $\boldsymbol{L}^{-1}[t, \omega_m]\,(\boldsymbol{R} + \omega_m \boldsymbol{G}[t, \omega_m])$, coefficient matrix of the differential state equation
s	= $(\omega - \omega_m)/\omega$, slip
T	= torque (N–m)
t	= time (sec)
Δt	= stepwidth (sec)
$\boldsymbol{V}[t]$	= $\{v_a, v_b, v_c, v_d, v_e, v_f\}$, voltage vector
$\boldsymbol{Z}[t, \omega_m]$	= $\boldsymbol{R} + \omega_m \boldsymbol{G}[t, \omega_m] + \boldsymbol{L}[t, \omega_m]\,(\mathrm{d}/\mathrm{d}t)$, impedance matrix
ω	= impressed voltage source angular velocity (rad/sec)
ω_m	= mechanical angular velocity transformed into electrical angular velocity (rad/sec)

Subscripts a, b, c refer to stator branch quantities, d, e, f refer to rotor branch quantities, i and j refer respectively to the row and column in an inductance matrix.

1. Introduction

Among all types of alternating current motors the one of induction type is by far the most popular and is used very widely. It is quite often designed for use on a polyphase circuit (usually three-phase) over the whole horsepower range. A polyphase induction motor has many excellent characteristics, such as the simpleness of its structure, its inherent self-starting character and the high reliability in its behavior. This machine is equipped with both a primary winding (usually stator) and a secondary winding (usually rotor) as shown in fig. 1. In normal use an energy source is connected to one winding alone, the primary winding.

Currents are made to flow in the secondary winding by induction, thereby creating an ampere-conductor distribution that interacts with the primary magnetic field distribution and produces a unidirectional torque.

At the starting time (or when controlled by semiconductor elements) we choose certain combinations of stator voltages, stator resistances and rotor resistances. By these choices we classify

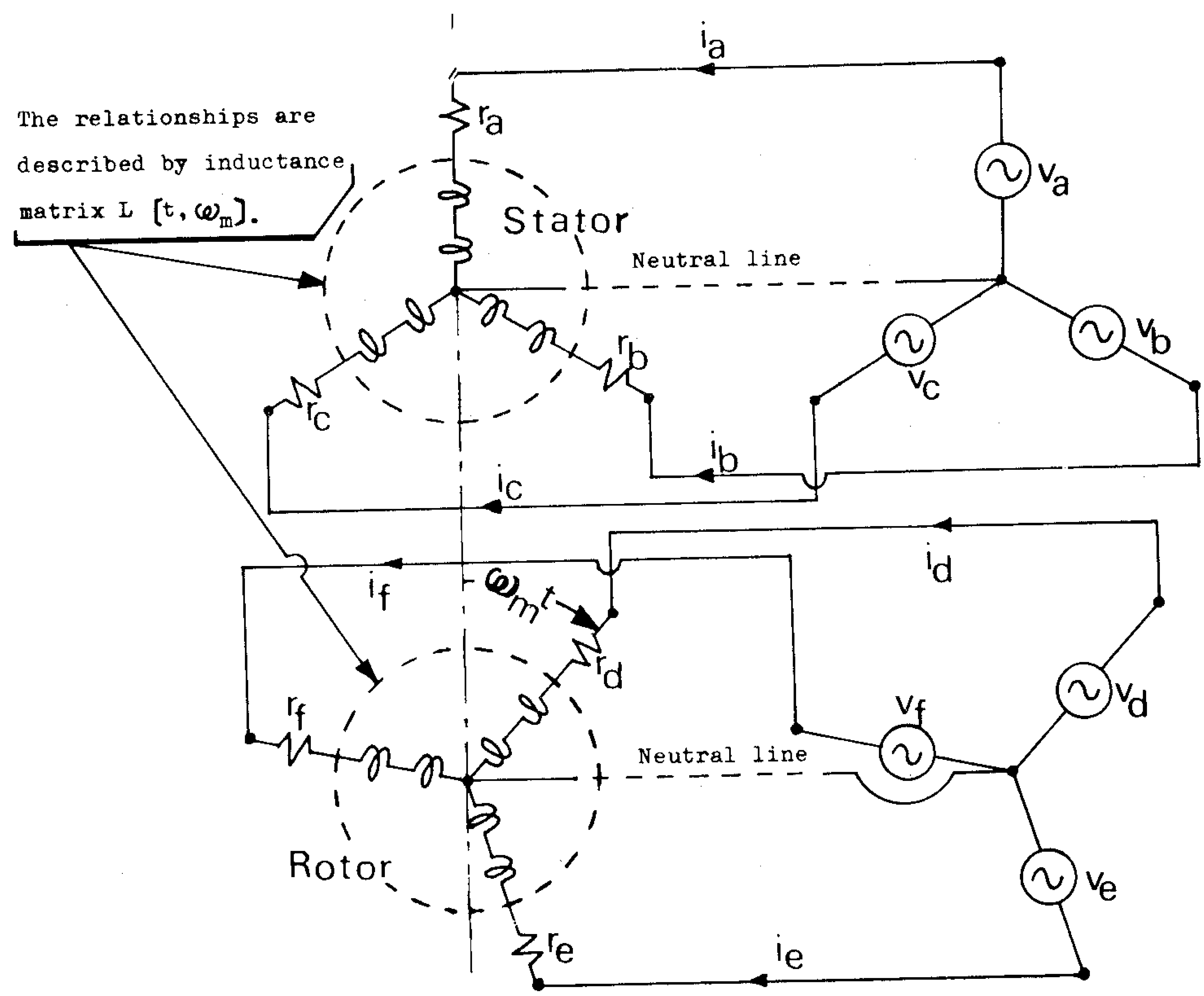


Fig. 1. Circuit diagram of the three-phase induction motor.

two cases – "balanced conditions" and "unbalanced conditions" – which are explained in detail in the Appendix.

In the many theoretical studies (e.g. [1]–[9]) which have been made for transient and steady states in polyphase induction motors it has been conventional to use very complex and tedious tensor transformations. In this paper we avoid these transformations by directly employing numerical methods to solve the fundamental equations of the three-phase induction motor for balanced and unbalanced conditions. The results are compared with those of conventional tensor transformation methods and also with experimental results.

2. Fundamental equations

The circuit diagram of the three-phase induction motor is shown in fig. 1 and is composed of six main branches, three stator and three rotor.

The set of Kirchhoff's equations for the circuit system consists of six simultaneous differential equations and is preferably expressed as a matrix equation between a voltage vector $\boldsymbol{V}[t]$ and a current vector $\boldsymbol{I}[t]$. The first three components of $\boldsymbol{V}[t]$ – v_a, v_b, v_c – are the impressed voltages of the stator branches, while the remaining three – v_d, v_e, v_f – are those of the rotor branches. The components of $\boldsymbol{I}[t]$ are similarly indexed. With $\boldsymbol{Z}[t, \omega_m]$ denoting the impedance matrix, the fundamental equation is given as

$$\boldsymbol{V}[t] = \boldsymbol{Z}[t, \omega_m]\, \boldsymbol{I}[t] \,, \tag{1}$$

where the quantity ω_m is the mechanical angular velocity.

The torque T of the motor is given in terms of the number of pole pairs p, the current matrix $\boldsymbol{I}[t]$ and the torque matrix $\boldsymbol{G}[t, \omega_m]$ (whose meaning will be given later):

$$T = (p/2)\boldsymbol{I}^{t}[t]\, \boldsymbol{G}[t, \omega_m]\, \boldsymbol{I}[t] \,. \tag{2}$$

The impedance matrix $\boldsymbol{Z}[t, \omega_m]$ is given in terms of three matrices – the resistance matrix $\boldsymbol{R}$, the torque matrix $\boldsymbol{G}[t, \omega_m]$ and the inductance matrix $\boldsymbol{L}[t, \omega_m]$:

$$\boldsymbol{Z}[t, \omega_m] = \boldsymbol{R} + \omega_m \boldsymbol{G}[t, \omega_m] + \boldsymbol{L}[t, \omega_m]\, (\mathrm{d}/\mathrm{d}t) \,. \tag{3}$$

The resistance matrix $\boldsymbol{R}$ is a diagonal matrix. The first three diagonal elements – r_a, r_b, r_c – are the resistances of the three stator branches, while the remaining three – r_d, r_e, r_f – are those of the rotor branches.

The inductance matrix is

$$L[t, \omega_m] = l + L' + M[t, \omega_m]$$

$$= \begin{bmatrix} l_a+L_{11} & L_{12}\cos(2\pi/3) & L_{13}\cos(4\pi/3) & M_{14}\cos(\omega_m t) & M_{15}\cos(\omega_m t+2\pi/3) & M_{16}\cos(\omega_m t+4\pi/3) \\ & l_b+L_{22} & L_{23}\cos(2\pi/3) & M_{24}\cos(\omega_m t-2\pi/3) & M_{25}\cos(\omega_m t) & M_{26}\cos(\omega_m t+2\pi/3) \\ & & l_c+L_{33} & M_{34}\cos(\omega_m t-4\pi/3) & M_{35}\cos(\omega_m t-2\pi/3) & M_{36}\cos(\omega_m t) \\ & & & l_d+L_{44} & L_{45}\cos(2\pi/3) & L_{46}\cos(4\pi/3) \\ & \text{SYMMETRICAL} & & & l_e+L_{55} & L_{56}\cos(2\pi/3) \\ & & & & & l_f+L_{66} \end{bmatrix}, \quad (4)$$

where l is the leakage inductance matrix, L' is the self-inductance matrix and $M[t, \omega_m]$ is the mutual inductance matrix; also the L_{ij} and M_{ij} are (ordinary) self-inductance and mutual inductance coefficients, respectively.

The torque matrix $G[t, \omega_m]$ is defined as

$$G[t, \omega_m] = (1/\omega_m)(\mathrm{d}/\mathrm{d}t) L[t, \omega_m] . \quad (5)$$

In order to solve eq. (1), it is necessary to take the following facts into consideration. When balanced conditions are satisfied, the last three (rotor) components of the voltage vector $V[t]$ are always zero, viz. $v_d = v_e = v_f = 0$, where the first three (stator) components satisfy the relation $v_a + v_b + v_c = 0$, as shown in the Appendix.

Also, when balanced conditions are satisfied or the stator components and the rotor components of the current vector $I[t]$ satisfy respective the relations $i_a + i_b + i_c = 0$ and $i_d + i_e + i_f = 0$.

Since the mechanical angular velocity ω_m is smaller than, or at most equal to, the angular velocity ω of the stator impressed voltage in the actual operation of the three-phase induction motor without space harmonics [10], the time variations of the matrix elements in the inductance matrix $L[t. \omega_m]$ are slow and at most equal to the time variations of the stator impressed voltages.

Therefore, we assume that the elements in $L[t, \omega_m]$ take constant values in the time interval from t to $t + \Delta t$. Thus, the fundamental equations of the three-phase induction motor reduce to linear simultaneous differential equations with constant coefficeints during the interval from t to $t + \Delta t$.

Now that eq. (1) can be taken as a system of linear simultaneous differential equations with constant coefficients, the difference method is applicable to solving it numerically.

By considering eq. (3), eq. (1) can be rewritten as

$$(\mathrm{d}/\mathrm{d}t) I[t] = -S[t, \omega_m] I[t] + L^{-1}[t, \omega_m] \; V[t] \quad (6)$$

where

$$S[t, \omega_m] = L^{-1}[t, \omega_m](R + \omega_m G[t, \omega_m]) . \quad (7)$$

With the coefficients evaluated at $t + \Delta t$, the integral of eq. (6) from t to $t + \Delta t$ is

$$\boldsymbol{I}[t + \Delta t] = \exp(-\Delta t\boldsymbol{S}[t + \Delta t, \omega_m])\boldsymbol{I}[t] + \{\boldsymbol{I}_\sigma - \exp(-\Delta t\boldsymbol{S}[t + \Delta t, \omega_m])\}\{\boldsymbol{R} + \omega_m \boldsymbol{G}[t + \Delta t, \omega_m]\}^{-1} \boldsymbol{V}[t + \Delta t] , \quad (8)$$

where $\boldsymbol{I}_\sigma$ denotes the unit matrix of order 6. This is used as a finite difference equation to solve eq. (6), with the exponential function approximated by

$$\exp(-\Delta t\boldsymbol{S}[t + \Delta t, \omega_m]) = \{\boldsymbol{I}_\sigma + A\,\Delta t\boldsymbol{S}[t + \Delta t, \omega_m]\}^{-1}\{\boldsymbol{I}_\sigma(1 - A)\Delta t\boldsymbol{S}[t + \Delta t, \omega_m]\} . \quad (9)$$

Particular values of the parameter A yield the forward difference method ($A = 0$) the central difference method ($A = 0.5$), and the backward difference method ($A = 1$) (see e.g. [11, ch. 8]). In this paper, practical computations are carried out with $A = 0.5$ and compared with results computed with other values of A as discussed in section 3.

Eq. (1) is directly solved by the numerical method for balanced conditions because no currents flow in the neutral line if the neutral is connected. However, the six relationships implied in eq. (1) are excessive for the majority of situations, and it is desirable to reduce the number of independent variables. Since we are dealing with star-connected machines with no neutral line as shown in fig. 1, the currents i_c and i_f can be eliminated by use of the relationship (see Appendix)

$$\boldsymbol{I}[t] = \boldsymbol{C}\boldsymbol{I}'[t] , \quad \text{i.e.} \quad \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} , \quad (10)$$

where $\boldsymbol{I}'[t]$ is the new current vector, and $\boldsymbol{C}$ is called the current connection matrix. Eqs. (1) and (2) are then transformed to

$$\boldsymbol{C}^{\mathrm{t}}\boldsymbol{V}[t] = \boldsymbol{C}^{\mathrm{t}}\boldsymbol{Z}[t, \omega_m]\,\boldsymbol{C}\boldsymbol{I}'[t] ,$$

$$T = (p/2)\boldsymbol{I}'^{\mathrm{t}}[t]\,\boldsymbol{C}^{\mathrm{t}}\boldsymbol{G}[t, \omega_m]\,\boldsymbol{C}\boldsymbol{I}'[t] . \quad (11)$$

The transient and steady state characteristics of the three-phase induction motor for balenced and unbalanced conditions are computed by eq. (11), using the procedures of eq. (8) and eq. (9).

3. Numerical solutions

The various constants of a motor used in calculation are listed in table 1 for balanced conditions, in table 2 for unbalanced conditions, and the constants of the actual experimented motor are listed in table 3.

Table 1
Various constants of the calculated motor for balanced conditions

	$\omega = 100\pi$ (rad/sec)	
Voltages	$v_a = \sqrt{2/3}\ 200 \sin(\omega t)$	(v)
	$v_b = \sqrt{2/3}\ 200 \sin(\omega t - 2\pi/3)$	(v)
	$v_c = \sqrt{2/3}\ 200 \sin(\omega t - 4\pi/3)$	(v)
	$v_d = v_e = v_f = 0$	(v)
	Initial currents are all zero	
	Stepwidth $\Delta t = 0.00005$ (sec)	
	Number of pole pairs $p = 2$	
Resistances	$r_a = r_b = r_c = 1.13$	(Ω)
	$r_d = r_e = r_f = 1.25$	(Ω)
Inductances	$L_{11} = L_{22} = L_{33} = L_{12} = L_{13} = L_{23} = 0.11466$	(H)
	$L_{44} = L_{55} = L_{66} = L_{45} = L_{46} = L_{56} = 0.11466$	(H)
	$M_{14} = M_{15} = M_{16} = M_{24} = M_{25} = 0.109$	(H)
	$M_{26} = M_{34} = M_{35} = M_{36} = 0.109$	(H)
	$l_a = l_b = l_c = l_d = l_e = l_f = 0.00533$	(H)

Table 2
Various constants of the calculated motor for unbalanced conditions

(a) Unbalanced stator impressed voltage	$v_a = \sqrt{2/3}\ 200 \sin(\omega t)$, $v_b = v_c = 0$ The other constants are the same as in table 1	(v)
(b) Unbalanced stator resistance	$r_a = 10.0$ The other constants are the same as in table 1	(Ω)
(c) Unbalanced rotor resistance	$r_d = 10.0$ The other constants are the same as in table 1	(Ω)

For the comparison, we solve numerically the simultaneous differential equations linearized by the conventional tensor transformation methods [4]–[9] by using the backward difference method, central difference method and forward difference method. Among the results obtained by each method, there are only small differences. These results also agree fairly well with our numerical solutions of eq. (1) or eq. (11) for balanced conditions computed by the central difference method. Rigorous analytical solutions are obtained for balanced conditions by the conventional revolving

Table 3
Various constants of the experimented motor for balanced and unbalanced conditions

Voltages	$\omega = 100\pi$	(rad/sec)
	$v_a = \sqrt{2/3}\ 200 \sin(\omega t)$	(v)
	$v_b = \sqrt{2/3}\ 200 \sin(\omega t - 2\pi/3)$	(v)
	$v_c = \sqrt{2/3}\ 200 \sin(\omega t - 4\pi/3)$	(v)
	$v_d = v_e = v_f = 0$	(v)
	Stepwidth $\Delta t = 0.00005$	(sec)
	Number of pole pairs $p = 2$	
Resistances	$r_a = r_b = r_c = 10.835$	(Ω)
	$r_d = r_e = r_f = 1.0$	(Ω)
Inductances	$L_{11} = L_{22} = L_{33} = L_{12} = L_{13} = L_{23} = 0.245$	(H)
	$L_{44} = L_{55} = L_{66} = L_{45} = L_{46} = L_{56} = 0.0369$	(H)
	$M_{14} = M_{15} = M_{16} = M_{24} = M_{25} = 0.0952$	(H)
	$M_{26} = M_{34} = M_{35} = M_{36} = 0.0952$	(H)
	$l_a = l_b = l_c = 0.02119$	(H)
	$l_d = l_e = l_f = 0.003194$	(H)
Unbalanced stator resistance	$r_a = 30.835$	(Ω)
	The other constants are the same as in the above table	

field theory [4]–[9]. Numerical solutions by our method reproduce these rigorous solutions within discrepancies of a few percent.

The numerical solutions of eq. (1) or eq. (11) for balanced conditions computed by the backward difference method were somewhat small compared with the results obtained by the central difference method. On the contrary, the forward difference method yielded larger results. Therefore, we adopted the central difference method for the digital simulation of a polyphase induction motor.

Some examples of numerical solutions of eq. (11) for balanced conditions computed by the central difference method are shown in fig. 2. The steady state numerical solutions of eq. (11) for balanced conditions computed by the central difference method are shown together with the above mentioned rigorous solutions in fig. 3. Some examples of numerical solutions of eq. (11) for unbalanced conditions computed by the central difference method are shown in fig. 4, and the comparisons of the steady state experimental and computational results are shown in fig. 5.

4. Conclusion

One of the merits of our direct integral method is that mathematical treatments are common both for balanced conditions and for any unbalanced conditions, while the conventional tensor transformations are confronted with serious difficulties in obtaining a linearized model for un-

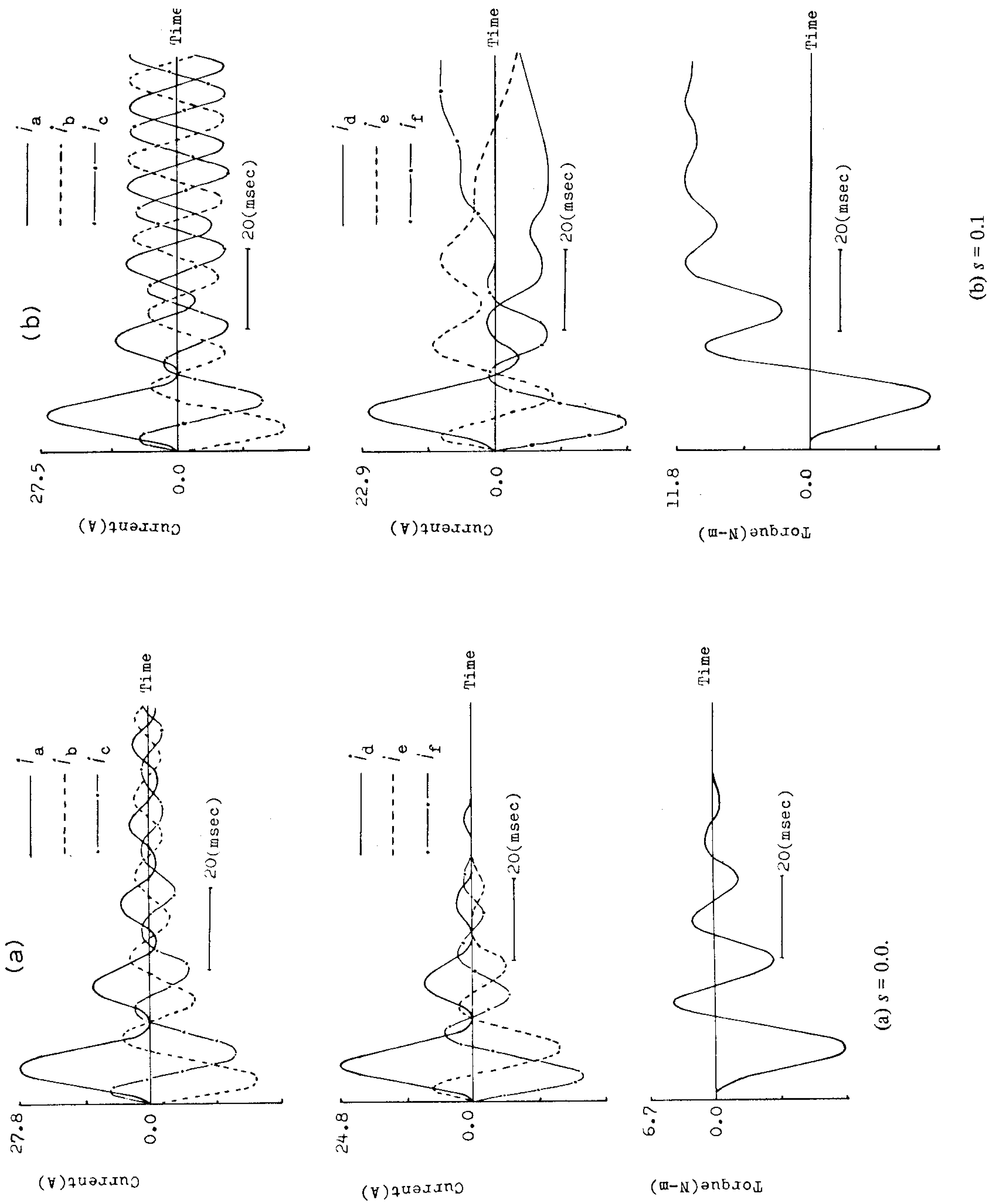
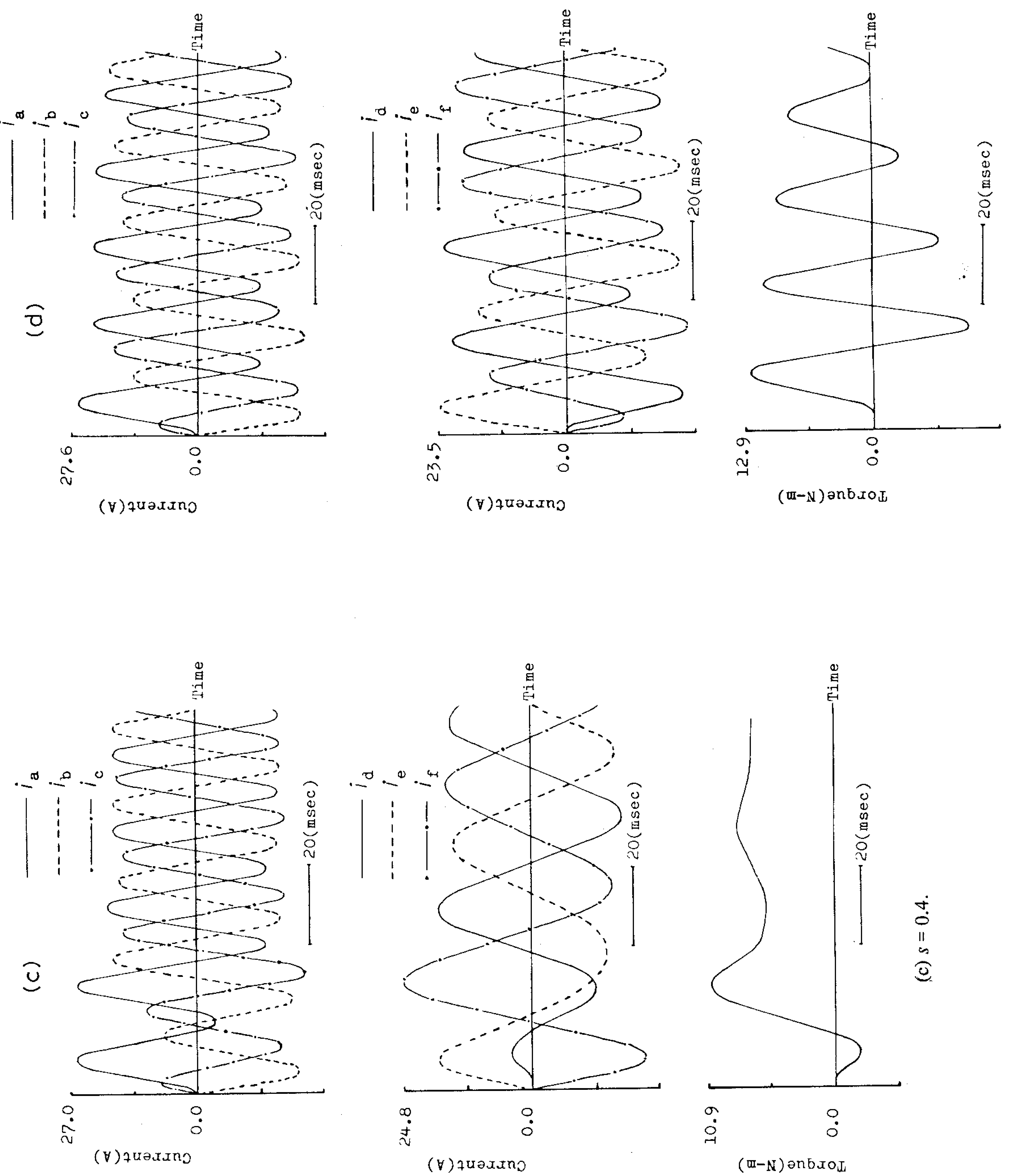


Fig. 2. Numerical examples for balanced conditions.



(c) $s = 0.4$.

(d) $s = 0.8$.

Fig. 2. (continued)

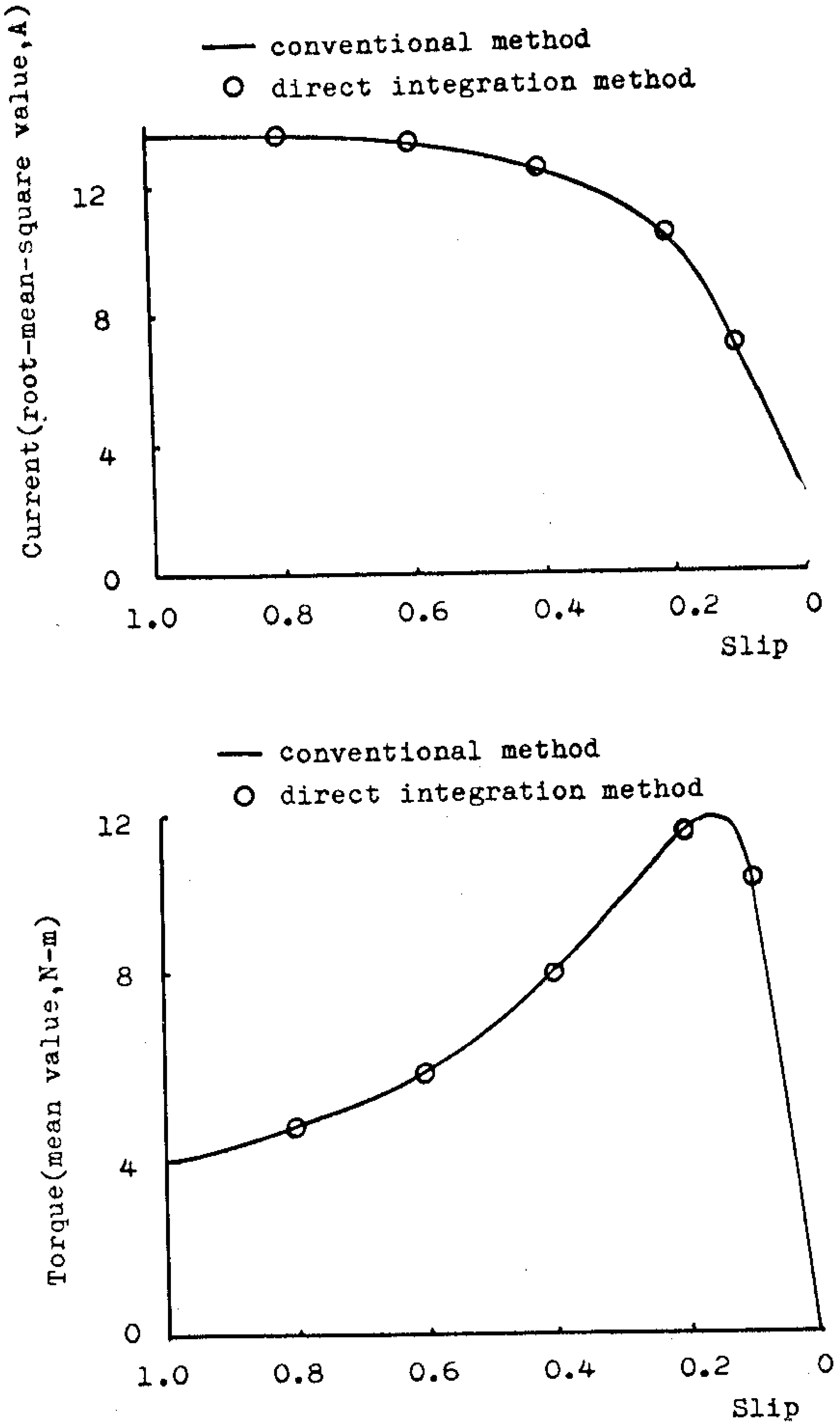


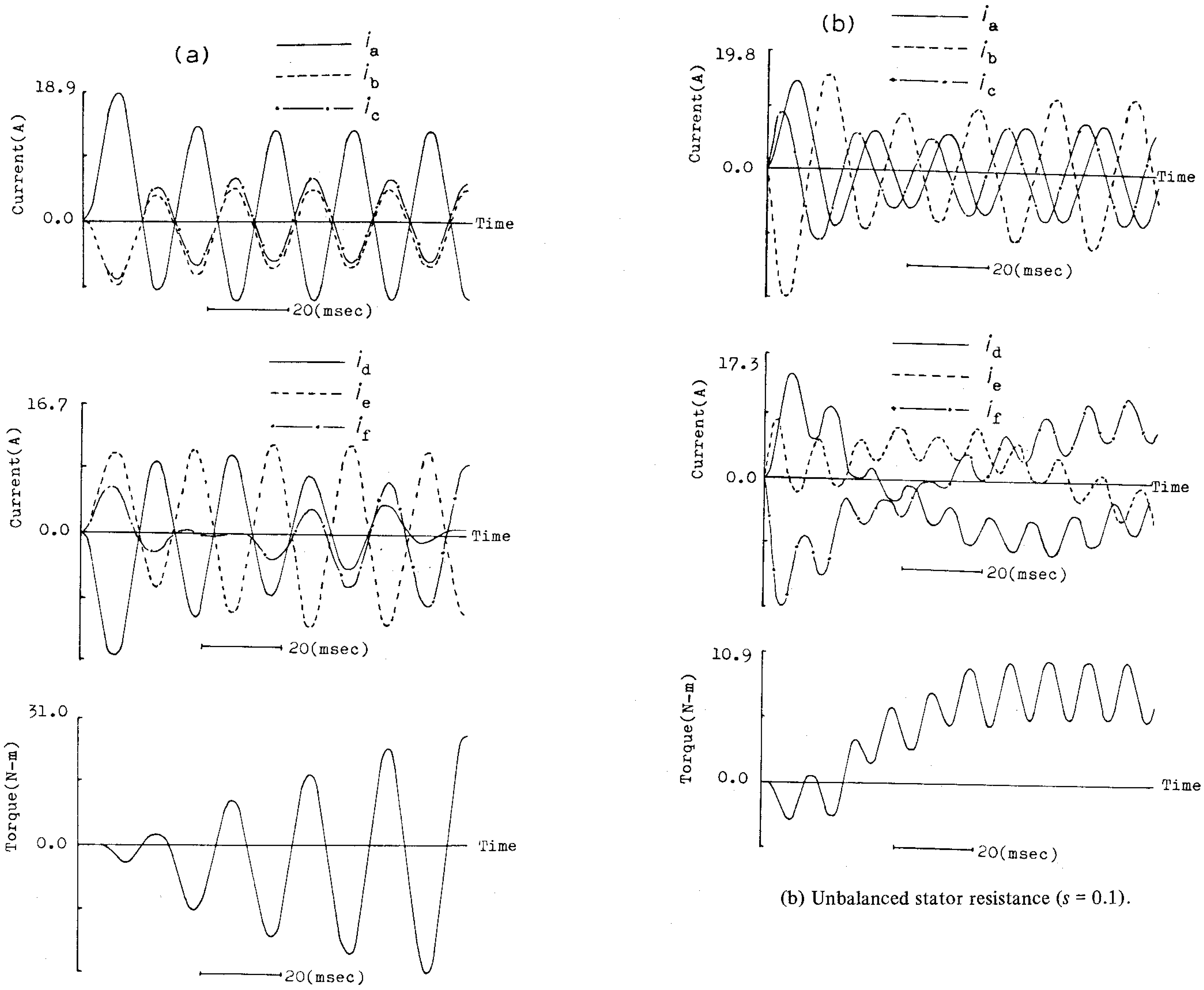
Fig. 3. Numerical examples of the steady state characteristics compared with the values that are computed by the conventional revolving field theory (each stator current i_a, i_b, i_c and rotor current i_d, i_e, i_f represented by the root mean square value are the same values respectively).

balanced conditions. So the method proposed here is not only useful for the analysis of the polyphase induction motor, but may be applicable to any other alternating current machines.

The algorithm is so simple that a computer program of our direct integration method can be easily written down directly from the fundamental differential equations without any other manual work and can be used for any unbalanced conditions. Thus a program of considerable generality is obtained with little programming effort.

If the relevant stepwidth Δt is chosen, then the numerical solutions obtained by our direct integration method have enough accuracy for engineering problems. In this paper, sufficiently accurate solutions are obtained by the selection of a stepwidth Δt which is sufficiently small to obtain the correct wave forms of the stator impressed voltages.

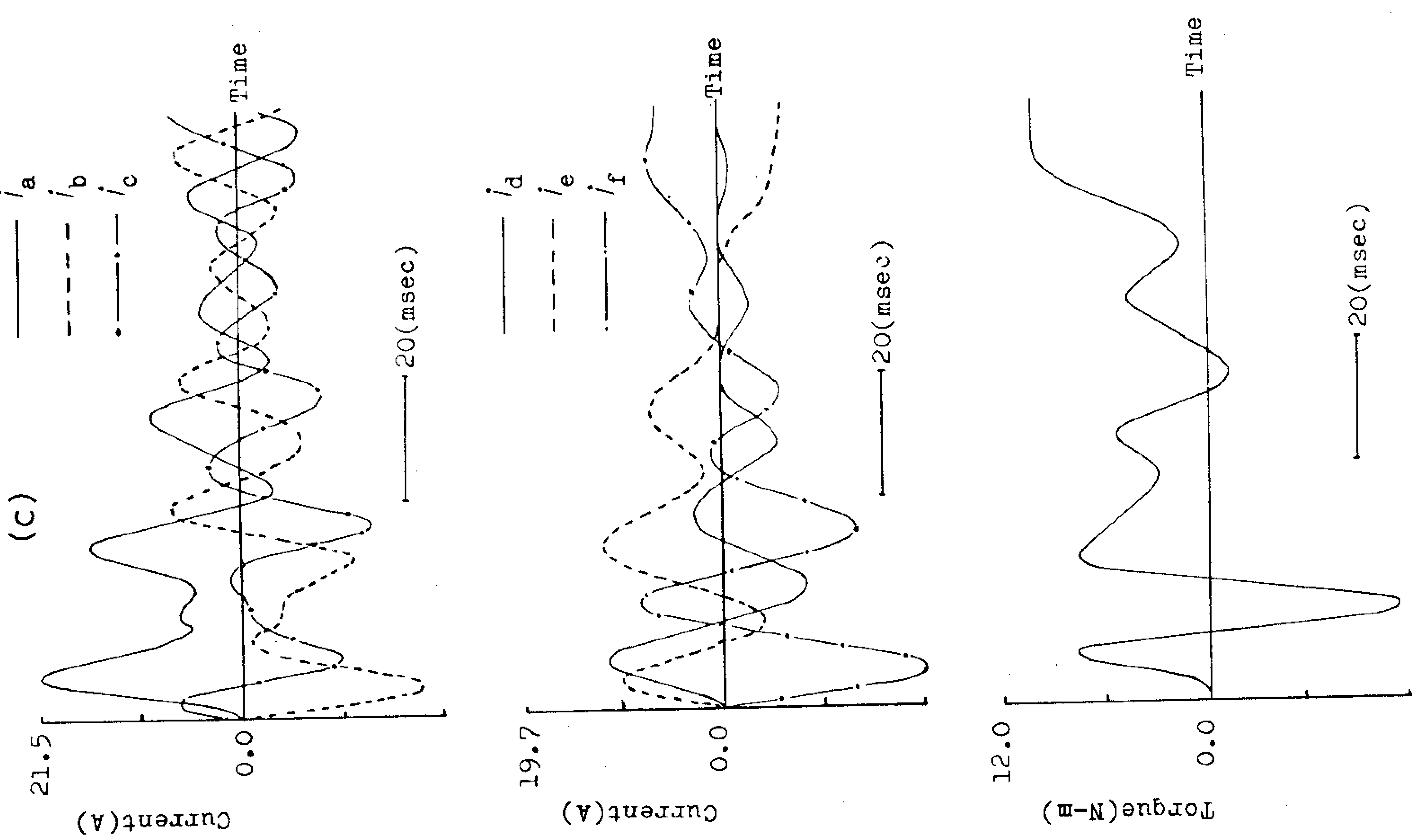
Finally, we have recently proposed a quite effective digital simulation method of polyphase induction motors. We wish to apply the direct integration method to the problem of a polyphase



(a) Unbalanced stator impressed voltage (s = 0.95).

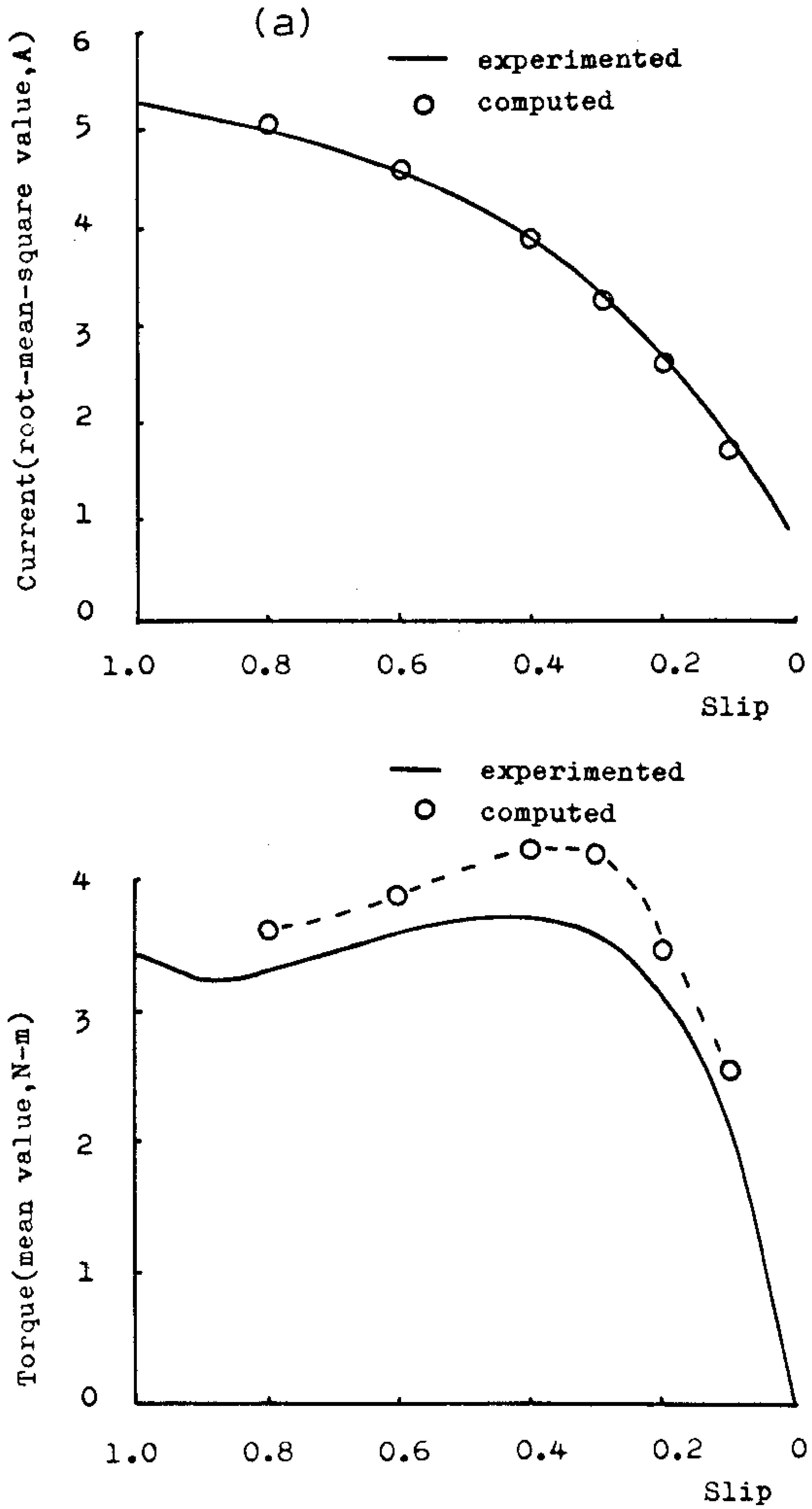
(b) Unbalanced stator resistance (s = 0.1).

Fig. 4. Numerical examples for unbalanced conditions.

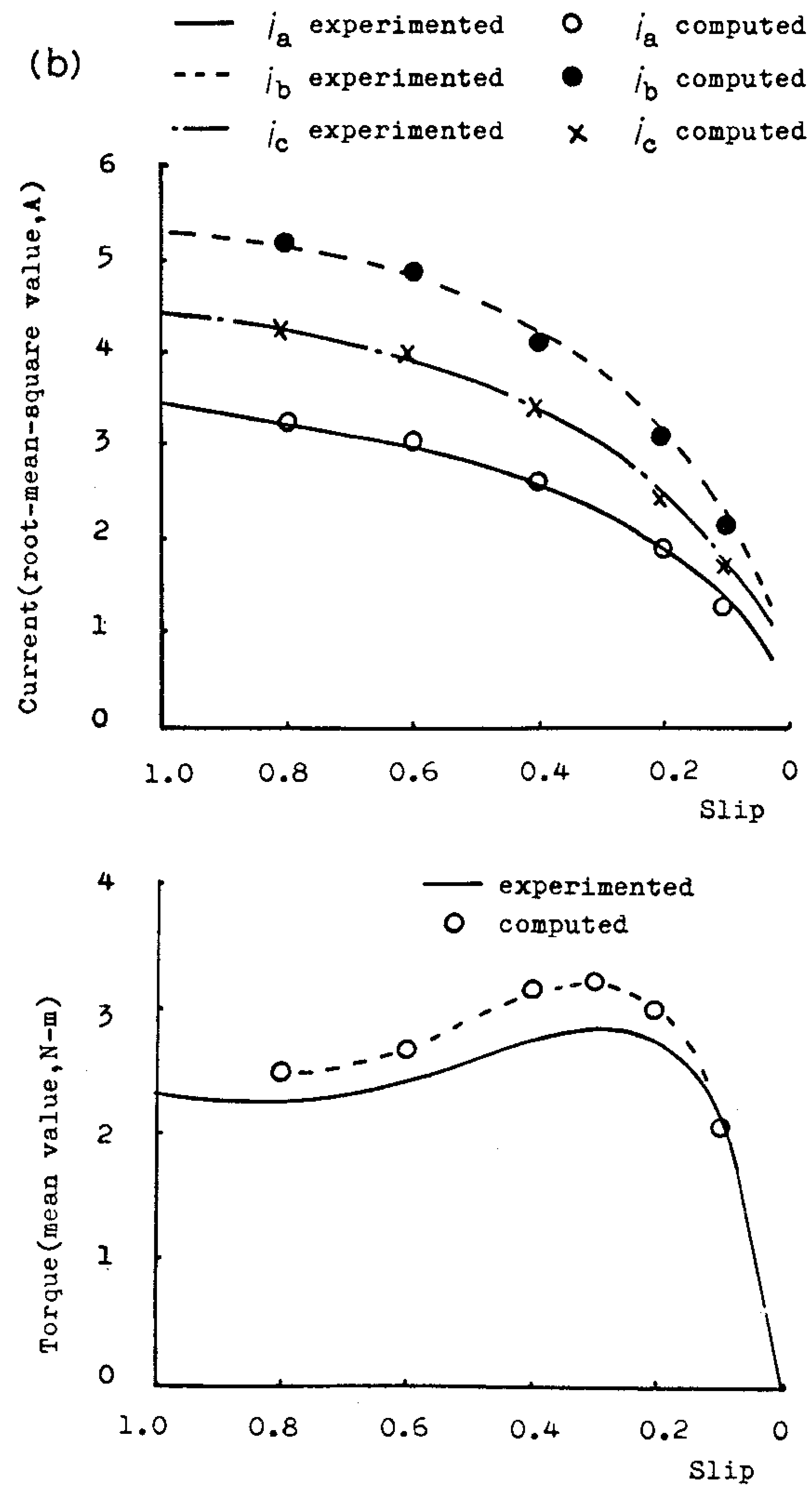


(c) Unbalanced rotor resistance ($s = 0.1$).

Fig. 4. (continued).



(a) Balanced conditions (each stator current i_a, i_b, i_c and rotor current i_d, i_e, i_f represented by the root mean square value are the same values respectively).



(b) Unbalanced stator resistance.

Fig. 5. Comparison of the steady state experimental and computational results.

induction motor supplied with nonsinusoidal waves as the stator impressed voltage. One of the authors (Saito) now intends to apply the method to the polyphase induction motor with space harmonics [10].

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The practical computations given in this paper were carried out by using the computers FACOM 230–45S of Hosei University and HITAC 8800 of Tokyo University.

Appendix: Balanced and unbalanced conditions

If the conditions that the stator impressed voltages v_a, v_b, v_c have the same amplitude, same angular velocity and relative phase difference $2\pi/3$ (in the case of three-phase voltage) are satisfied, then such a case is called "balanced stator impressed voltage". The case in which one or more of these conditions are not satisfied is called "unbalanced stator impressed voltage".

If the conditions shown in table 4 are satisfied, then such a case is called "balanced impedance". The case in which one or more of these conditions are not satisfied is called "unbalanced impedance". In particular, if the stator (rotor) resistances are not all the same, then such a case is called "unbalanced stator (rotor) resistance" respectively.

Table 4

	stator	rotor
resistances	$v_a = v_b = v_c$	$v_d = v_e = v_f$
leakage inductances	$l_a = l_b = l_c$	$l_d = l_e = l_f$
self-inductances	$L_{11} = L_{12} = ... = L_{33}$	$L_{44}, L_{55} = ... = L_{66}$
Mutual inductances	$M_{11} = M_{15} =$	$= M_{36}$

In general, if the conditions of balanced stator impressed voltage and balanced impedance are satisfied, then such a case is called "balanced conditions". The case in which one or more of these conditions of balanced stator impressed voltage and balanced impedance are not satisfied, then such a case is called "unbalanced conditions".

In balanced conditions, from the conditions of balanced stator impressed voltage, the following relationship is established:

$$v_a + v_b + v_c = 0 \,. \tag{A.1}$$

The examples of eq. (A.1) are shown in table 1 and table 3, and from the conditions of balanced impedance (e.g. see table 1) the following relationships are established:

$$i_a + i_b + i_c = 0 \,, \tag{A.2}$$

$$i_d + i_e + i_f = 0 \,. \tag{A.3}$$

The examples which satisfy eqs. (A.2) and (A.3) are as follows:

$$i_a = I_s \sin(\omega t - h) \,,$$

$$i_b = I_s \sin(\omega t - h - 2\pi/3) \,,$$

$$i_c = I_s \sin(\omega t - h - 4\pi/3) \,,$$

$$i_d = I_r \sin(s\omega t - k) \,,$$

$$i_e = I_r \sin(s\omega t - k - 2\pi/3) \,,$$

$$i_f = I_r \sin(s\omega t - k - 4\pi/3) \,,$$

where I_s, I_r, h and k are constants.

Therefore, in balanced conditions, if the neutral line is connected or disconnected, the same numerical solutions can be obtained from eq. (1) (which is written as it were equipped with the neutral line) in section 2.

In unbalanced conditions, especially in unbalanced stator impressed voltage, the relationship (A.1) is not established, and if the neutral line is disconnected (star-connected machine as shown in fig. 1), then the previosuly described relationships (A.2) and (A.3) must be established.

Then, in unbalanced conditions, the relationships (A.2) and (A.3) must be introduced in the fundamental equations described by eq. (1) in section 2. Therefore, an arbitrary one of the independent variables of eq. (A.2) and eq. (A.3) is represented by the remaining terms of eq. (A.2) and eq. (A.3) respectively, and their relationships are described by the current connection matrix $\boldsymbol{C}$ [in this paper, current i_c in eq. (A.2) and current i_f in eq. (A.3) are represented by the remaining currents in eq. (A.2) and eq. (A.3) respectively, as described by eq. (10) and eq. (11) in section 2], which is introduced to the fundamental equations and torque equation as described by the procedures of eq. (11).

However, if the neutral line is connected, then the relationships (A.2) and (A.3) are not satisfied, in this case, numerical solutions are obtained by the eq. (1) in section 2 without any transformations [eq. (11) in section 2].

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