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Effect of Different Orthogonal Wavelet Basis on Multiresolution Image Analysis of a Turbulent Flow

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Abstract: In order to extract the coherent structure and the most essential scales governing turbulence, the best choice of orthogonal wavelet basis to use for the multiresolution image analysis of the turbulent structures was investigated in this paper. the digital imaging photograph of jet slice was decomposed by two-dimensional orthogonal transform based on Daubechies, Coifman and Baylkin bases. It is found that these orthonormal wavelet families with index N<10 were inappropriate for multiresolution image analysis of turbulent flow. The multiresolution images of turbulent structures were very similar when using the wavelet basis with the higher index number, even though wavelet basis is different function. From the image components in orthogonal spaces with different scales, the further evident of the multiscale structures in jet can be observed, and the edges of the vortices at different resolutions or scales and the coherent structure can be easily extracted.

Keywords: Coherent Structure, Jet, Multiresolution Image Processing, Orthogonal Wavelet Basis, Orthogonal Wavelet Transform, Vortex

1. Introduction

Coherent structures are known to exist and are responsible for most of the momentum transfer in turbulent jets. Many identification techniques, such as image processing, spectra analysis, spatial correlation functions, education schemes, proper orthogonal decomposition, stochastic estimation, pattern recognition, and wavelet transform, are well established to determine coherent structures. However, the local scales with respect to space-time change continuously for the turbulence and the coherent structure in both time and scale spaces has not yet been clarified. Identification of coherent structure requires the acquisition of detailed quantitative data on such structure characteristics as location, size, strength, etc. Until now, traditional analytical techniques cannot give us sufficient or detail information. Laufer [(1)] (1975) pointed that the visualization of organized motions in shear layers showed that the conditional sampling measurement had been hiding very important features of turbulence. To solve these problems, we should develop more powerful identification or analytical techniques. In the present study we investigate the development of a new identification technique, called two-dimensional orthogonal wavelets, to analyze full field measurements or image process in turbulent shear flow, and focus on the application of wavelets to the experimental study of jet.

In the limited open literature available, Everson et al. (1990) analyzed two-dimensional dye concentration data from a turbulent jet using the continuous wavelet transform, and revealed the nature and self-similarity of the inner structure of the jet. Lewalle et al. (1994) applied the wavelet transform to velocity signals in the inner mixing layer of a coaxial jet and analyzed the dominance of non-periodic vortices at a given time scale. Li and Nozaki (1995) displayed very different scale eddies, the breakdown of a large eddy and the successive branching of a large eddy structure in a plane turbulent jet by analyzing the velocity signals at various positions with the wavelet transform. A wavelet decomposition of fluctuating velocities in a planar jet was used to the Reynolds stress in scale space by Gordeyev and Thomas (1995). Using wavelet decomposition the unsteady aspects of the transition of a jet shear layer were studied by Gordeyev et al. (1995). Li (1997, 1998) proposed the wavelet correlation and time-frequency correlation methods based on wavelets, and applied to revealing the structure of eddy motion and coherent structure in the turbulent shear flow in both Fourier and physical spaces. Li (1997, 1998) also developed the wavelet spatial statistics using wavelets and analyzed the large-scale eddy motions and the structure of the energy transfer over a two-dimensional frequency-physics plane in a turbulent shear flow.

The large eddy structure of a bounded turbulent jet was first studied by applying the wavelet transform, local wavelet Reynolds stress and wavelet Reynolds stress to the experimental velocity signals in Li et al. (1997). Walker et al. (1997) studied experimentally the multiple acoustic modes and shear layer instability waves in a rectangular underexpanded jet using wavelet transform. The discrete wavelet transform using Daubechies orthogonal wavelet bases were also applied to analyzing velocity signals of a plane turbulent jet in the dimension of time and frequency by Li et al. (1998). Besides these application studies, several new tools and diagnostics based on the wavelet transform, such as wavelet intermittency, wavelet Reynolds number, wavelet spectrum, wavelet cross spectrum, and wavelet correlation function were developed. They offered the potentials extracting new information from various flow fields. However, most of above researches limited to the analysis of jet structure using the one-dimensional wavelet transform. To gain deeper insight into multi-scale structures and extract coherent structure in turbulent flows, it is an important to analyzing the full field by image processing. Although there were several researches (e.g., Everson et al., 1990; Brasseur and Wang, 1992; Spedding et al., 1993; Dallard and Browand, 1993; Dallard and Spedding, 1993), who applied two or three-dimensional wavelet transform to full field measurements or simulation data, they have thus far concerned the continuous wavelet transform. The coefficients of continuous wavelet transform can extract the characterization of local regularity, but it is unable to reconstruct the original function because the mother wavelet function is non-orthogonal function. In the image processing it is importance to reconstruct the original image from wavelet composition and to study multiresolution image in the range of various scales.

One of the dominate principal analytical tool in the image processing is Fourier analysis, which can be used to convert point data into a form that is useful for analyzing frequencies. In some problems, however, Fourier analytic techniques are inadequate or lead to extremely onerous computations. A case is that each Fourier coefficient contains complete information about the behavior of images at one scale of frequency but no information about its behavior at other scales or frequencies. In contract, many applications require an analysis of the series at broader scale. For example, a region of rapid change in the series can only be detected by examining many points at once. A wavelet is a bandpass filter with the additional space location capability.

In this paper, the best choice of orthogonal wavelet basis to use for extracting information on the turbulent structures from the multiresolution image, which is obtained by the digital imaging photograph of two-dimensional orthogonal transform, is investigated.

2. Definition of Two-Dimensional Orthogonal Wavelets

In the previous papers (Li, 1997, 1998), we described applications of the one-dimensional continuous and discrete wavelet transforms to the turbulent jet. In this section, we introduce definitions of two-dimensional wavelet transform before applying it to image analysis. Let us consider a two-dimensional scalar field $f(\vec{x})$ and isotropic mother wavelet $\psi(\vec{x})$ by treating $\vec{x}=(x_1,x_2)$ as vector. The family of wavelets $\psi_{\vec{b},a}(\vec{x})$, which is translated by position parameter $\vec{b}\in R^2$ ($\vec{b}=(b_1,b_2)$) and dilated by scale parameter $a\in R^+$, is written as

$$\psi_{\vec{b},a}(\vec{x})=\frac{1}{a}\psi\left(\frac{\vec{x}-\vec{b}}{a}\right), \tag{1}$$

where $\psi(\vec{x})=\psi(|\vec{x}|)$ and $\psi(\vec{x})$ satisfies the admissibility condition

$$C_\psi=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{|\hat{\psi}(\vec{\omega})|^2}{|\vec{\omega}|}d^2\vec{\omega}<\infty \ , \tag{2}$$

i.e., (1) compact support or sufficiently fast decay;

(2) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(\vec{x})d^2\vec{x}=0.$

Several functions, such as, 2-D Morelet wavelet, Halo wavelet, and others, are often used as mother wavelet in area of fluid mechanics.

The continuous wavelet transform of $f(\vec{x})$ can be defined as

$$\begin{aligned} Wf(\vec{b},a) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\vec{x})\psi_{\vec{b},a}(\vec{x})d^2\vec{x} \\ &= \frac{1}{a}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\vec{x})\psi\left(\frac{\vec{x}-\vec{b}}{a}\right)d^2\vec{x} \end{aligned}, \tag{3}$$

The coefficients of continuous wavelet transform $Wf(\vec{b},a)$ can be interpreted as the relative contribution of scale a to the scalar field $f(\vec{x})$ at position $\vec{b}$.

If the wavelet is admissible, the inversion formula is

$$f(\vec{x})=\frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}a^{-3}Wf(\vec{b},a)\psi_{\vec{b},a}(\vec{x})dad^2\vec{b} \\ =\frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}a^{-4}Wf(\vec{b},a)\psi\left(\frac{\vec{x}-\vec{b}}{a}\right)dad^2\vec{b}, \tag{4}$$

The two-dimensional continuous wavelet transform has proved to be useful in several applications in the area of fluid mechanics and extracted the characterization of local regularity. However, it is unable to reconstruct the original function and to carry multiresolution analysis because the digital image is known at only discrete pixel values (whose spatial resolution depends on the resolution of the camera) and the mother wavelet function also is non-orthogonal function. Therefore it is impossible to decompose image into a set of orthonormal basis functions and is unable to analyze its multiresolution structure.

Therefore, for actual image analysis, the discrete wavelet transform is preferred. In the discrete wavelet transform, the dilation parameter a and the translation parameter $\vec{b}$ both take only discrete values in Eq. (3). For a scale a we choose the integer (positive and negative) powers of one fixed dilation parameter $a_0>1$, i.e., a_0^m, and different values of m correspond to wavelets of different widths. It follows that the discretization of the translation parameter $\vec{b}$ should depend on m: narrow wavelets (high frequency) are translated by small steps in order to cover the whole field, while wider wavelets (lower frequency) are translated by large steps. Since the width of is proportion to a_0^m, we choose therefore to discretize $\vec{b}$ by $\vec{b}=\vec{n}b_0a_0^m$, where b_0 is fixed. Starting from wavelet basis $\psi_{m,n}(x)=a_0^{-m/2}\psi(a_0^{-m}x-nb_0)$ the corresponding discretely family of wavelets is simply to take the tensor product functions generated by two one-dimensional bases:

$$\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2)=\psi_{m_1,n_1}(x_1)\psi_{m_2,n_2}(x_2) \tag{5}$$

For some very special choices of $\psi(x)$ and a_0, b_0, the $\psi_{m,n}(x)$ constitutes an orthonormal basis. In particular, if we choose $a_0=2$, $b_0=1$, then there exist $\psi(x)$, with good physics-frequency localization properties, such that the

$$\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2)=2^{-(m_1+m_2)/2}\psi(2^{-m_1}x_1-n_1)\psi(2^{-m_2}x_2-n_2) \tag{6}$$

constitute an orthogonal basis. In this basis the two variables x_1 and x_2 are dilated separately. The oldest example of a function $\psi(x)$ for which the $\psi_{m,n}(x)$ constitutes an orthogonal basis is the Haar function, constructed long before the term "wavelet" was coined. In the last ten years, various orthogonal wavelet bases, e.g., Meyer basis, Daubechies basis, Coifman basis, Battle-Lemarie basis, Baylkin basis, spline basis, and others, have been constructed. They provide excellent localization properties in both physical and frequency spaces. It is well known that different wavelet basis functions will preferentially move, between scales, different characteristics of the target dataset. For example, use of the Harr basis function will emphasize discontinuities in the target dataset, and similarly, would enhance reconstruction of a discontinuous fine grid field from a sample set. Other wavelet basis functions, such as the Daubechies family, emphasize the smoothness of the examined data. In this study, we employ Daubechies, Coifman and Baylkin basis to analyze the flow image, and discuss the effect of different orthogonal wavelet basis on the multiresolution results.

Therefore, the two-dimensional discrete wavelet transform is given by

$$Wf_{m_1,n_1;m_2,n_2}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(\vec{x})\Psi_{m_1,n_1;m_2,n_2}(\vec{x})d^2\vec{x}. \tag{7}$$

The reconstruction of the original scalar field can be achieved by using

$$f(\vec{x})=\sum_{m_1}\sum_{m_2}\sum_{n_1}\sum_{n_2}Wf_{m_1,n_1;m_2,n_2}\Psi_{m_1,n_1;m_2,n_2}(\vec{x}) \tag{8}$$

The total energy of the scalar field is given by summing over all scales and components as following

$$\sum_{i,j}\left(f(x_1^i,x_2^j)\right)^2=\sum_{m_1}\sum_{m_2}\sum_{n_1}\sum_{n_2}\left(Wf_{m_1,n_1;m_2,n_2}\right)^2 \tag{10}$$

Mallat and Meyer formulated the theory of multiresolution analysis in the fall of 1986, in order to provide a natural framework for the understanding and construction of wavelet bases. The goal of the multiresolution analysis is to get a representation of a function that is written in a parsimonious manner as a sum of its essential components. That is, a parsimonious representation of a function will preserve the interesting features of the original function, but will express the function in terms of a relatively small set of coefficients. Thus overcoming limitations of the two-dimensional continuous wavelet transform that cannot reconstruct the original function.

In mathematics, the multiresolution analysis consists of a nested set of linear function spaces V_j with the resolution of functions in increasing with j. More precisely, the closed subspaces V_j satisfy

$$\cdots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \cdots \tag{11}$$

with

$$\overline{\bigcup_{j \in Z} V_j} = L^2(\Re^2) \quad \text{and} \quad \bigcap_{j \in Z} V_j = \{o\} \tag{12}$$

The basis functions for the subspaces V_j are called scaling functions of the multiresolution analysis.

For every $j \in Z$, define the wavelet spaces W_j to be the orthogonal complement of in V_{j-1} of V_j. We have

$$V_{j-1} = V_j \oplus W_j \tag{13}$$

and

$$W_j \perp W_{j'} \qquad \text{if } j \neq j'. \tag{14}$$

i.e., any function in V_{j-1} can be written as the sum of a unique function in V_j and a unique function in W_j. In $L^2(\Re^2)$, the orthogonal basis for W_j is the family of wavelets $\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2)$ that is defined in Eq.(6). Thus, $L^2(\Re^2)$ can be decomposed into mutually orthogonal subspaces, and can be written as

$$L^2(\Re^2) = \bigoplus_{j \in Z} W_j \tag{15}$$

The detail regarding the wavelet transform and multiresolution analysis can be found in many references (e.g., Daubechies, 1992).

3. Multiresolution Image Analysis of the Turbulent Jet

Since Mallat (1998) first applies multiresolution analysis to the field of image processing, researchers have been making widespread use of multiresolution analysis for several years. The major applications of wavelets consist of image compression, image editing, multiresolution processing, multiscale edge detection, texture discrimination, and others. In this paper, we focus on multiresolution analysis of turbulent structure. It is well known that an image often includes too much information for real time vision processing. Multiresolution algorithm process less image data by selecting the relevant details that are necessary to perform a particular recognition task. Coarse to fine searches processes first a low-resolution image and zoom selectively into finer scales information. The aim of this paper is to analyze image multi-scale structure of jet flow simultaneously by decomposing the original image into W_j approximation spaces.

It is well known that turbulent jets exhibit complex structure with a wide range of coexisting scales and a variety of shapes in the dynamics. To understand the turbulent mixing process, various measures of the isosurface geometry from image appeared from 1980's. Recently, Catrakis and Dimotakis (1996) reported two-dimensional, spatial measurements of the jet-fluid concentration field in liquid-phase, turbulent jets and included scalar-field measures and isoscalar measures that were found to be functions of Reynolds number were performed. Furthermore, they presented scale distributions and fractal dimensions measure by level sets of concentration, and also showed the shape complexity of irregular surface based on area-volume measure of image. In order to gain deeper insight into the multiscale structures and coherent structure, we apply multiresolution analysis to the digital-imaging photographs of turbulent jet that were experimentally obtained by Catrakis and Dimotakis (1996).

As described in Catrakis and Dimotakis' paper (1996), experiments were carried out in liquid-phase turbulent-jet flows, and images of slices, which relied on laser-induced fluorescence digital-imaging techniques, through the three-dimensional scalar filed of round momentum-driven turbulent jets were obtained. Transverse sections in the far field of the jet, at downstream position z/d=275 (jet-nozzle diameter d is 2.54mm), were recorded on a cryogenically cooled 1024x1024 pixels CCD camera.

In this paper, black-and-white images of jet slices are expressed in a numerical form as a function $f(x_1, x_2)$ over two dimensions in which the function value $f(x_1^0, x_2^0)$ represents the "gray scale" value of the image at the position or pixel values (x_0, y_0). The "gray scale" values are then normalized to one. Multiresolution analysis is the result of a two-step process. Images are first decomposed into wavelet components and their wavelet spectrums are obtained. Reconstruction or inverse discrete wavelet transform is then done at each scale, and image multiresolutions are visualized in W_j approximation spaces. It is evident that a sum of all image components in W_j approximation spaces can reconstruct the original image. In order to investigate the effect of different orthogonal wavelet basis on the multiresolution image, Daubechies, Coifman and Baylkin bases are used. These orthogonal wavelet bases have respectively their families that are defined by different index number or the number of wavelet's coefficients. As the index number, i.e., the number of wavelet's coefficients, increases the wavelet becomes smoother closer to a smoothly windowed harmonic function, and the wavelet's Fourier transform becomes increasingly compact, i.e., are compact in the frequency domain.

At first the original image of the jet-fluid concentration with Re=4.5x10^3 at downstream position z/d=275 is decomposed into six W_j approximation spaces based on the two-dimensional orthonormal wavelet transform with the Daubechies basis of index N=4. Results of multiresolution images with six levels are displayed in Fig.1. False-colors have been assigned to the scalar values of wavelet transform, and the highest concentration is displayed as a deep red and the lowest as purple. Blue in each signifies the zero value. The sum of six image components can reconstruct the original image. From this figure it is evident that the images from level 1 to 6 respectively show non-smoothness distribution and any information on the flow structure of the large and medium scales are hardly obtained. At levels 7 and 8, however, edges of flow region and vortex in the shear layer may be observed at some certain. This is because Daubechies basis with index N=4 is nowhere differentiable, although it is continuous and compact in the physical domain. Within the range of our study, it is appropriate to say that the orthonormal wavelet families (Daubechies, Coifman and Baylkin bases) with index N<10 are inappropriate for multiresolution image analysis of turbulent flow. Therefore, it is very important that orthonormal wavelet basis must be smoothness function in the multiresolution image analysis of turbulence.

Then the same original image of the jet-fluid concentration is analyzed by the two-dimensional orthogonal wavelet transform with the Daubechies family with index N=18 and 14. The images of multiresolution structures of the same turbulent jet slice are shown in Fig.2 and 3, respectively. These images provide further evident of complex multi-scale structures in turbulent jet and may easily extract important scales that dominate the flow structure. Because the frequency character of six W_j approximation spaces based on the Daubechies basis of index N=18 differs from that of the Daubechies family with index N=14, the multiresolution image even at same level shows the different structures. Of course, the sum of each six image components can also reconstruct the original image. In order to gain the frequency information on six W_j approximation spaces for these two bases, two-dimensional Fourier Transform is employed to analyze the multiresolution images of six levels.

The images based on the Daubechies basis with index N=18, as shown in Fig.2, display smoothness and evident multiresolution structures, since Daubechies basis of index N=18 is more smoothness and compact support in the frequency domain. From the image for the lowest level 1, which corresponds to the broader scale of a=27.0~107.5mm, the interior of the flow may be identified by the blue line. A large peak that contains three peaks can be clearly observed near the center of jet. These peaks imply that a large-scale structure consists of three vortices. They are the uppermost and energy-containing vortices. With the broader scale of a=14.3~27.0mm, as shown in the image of the level 2, a lot of stronger peaks mainly appear in the edge of flow region, and correspond to the positions of vortices at this scale range. These vortices are more active in the shear layer and dominate the turbulent mixing process, which are referred to as the coherent structure of the problem. As decreasing scale to a=7.2~14.3mm at levels 3, peaks mainly concentrate on islands or lakes (as described in Catrakis and Dimotakis (1996)) of the flow region edge. The distribution of peaks indicates that vortices also undertake the turbulent mixing process within this scale range in this region. When the broader scale reaching to a=3.6~7.2mm, as shown in the image of the level 4, edges of vortex within this scale range can be clearly observed, especially are obvious in the shear layer. As increasing resolution to a=1.8~3.6mm, a finer approximation of the original image can be obtained at level 5. A clear distribution of vortex edges with smaller-scale can be observed, which is the "zoom-in" the image of the level 4. That is an important feature of multiresolution analysis. Edges of the smallest-scale vortex in this problem can be observed everywhere in the interior of the flow. This means that the smallest-scale vortices exist in the whole flow field. The image of level 6 describes the distribution of fine streamlines with the broader scale of a=0.8~1.8mm. From above results, it is can say that the edges of the vortices at different resolutions or scales and the coherent structure may be easily extracted by wavelet multiresolution analysis.

In Fig.3, levels from 1 to 6 correspond to broader scale of a=54.0~107.5*mm*, 27.0~54.0*mm*, 14.3~27.0*mm*, 7.2~14.3*mm*, 3.6~7.2*mm* and 1.8~3.6*mm*, respectively, when using the Daubechies basis of index N=14. From the image of the lowest level 1, the interior of the flow can be identified by the blue line, and a large peak in the interior of the flow represents a larger scale vortex on the center of jet. At the level 2, the distribution of peaks indicates that vortexes with broader scale a=54.0~27.0*mm* distribute uniformly in the interior of the flow. These vortexes gain the energy from the large-scale vortexes and then transit it to the smaller-scale vortexes. The images at these two levels or large broader scale differ from that of Fig.2 since they have different the frequency character in W_j approximation spaces. However, from level 3 to 6 images of flow structure are very similar to that of Fig.2 from level 2 to 5, respectively. Since the Daubechies basis of index N=14 has lower resolution than that of the Daubechies basis of index N=18 in the frequency domain, the image with the broader scale of a=0.8~1.8*mm* can not be obtained. However, the Daubechies basis of index N=14 has higher resolution in the region of lower frequency, and may be employ to study the large-scale structure of turbulence.

At last, Coifman family with the higher index is employed to analyze the same original image of the jet-fluid concentration. It is found that the multiresolution images using Coifman bases of index $N \geq 18$ show almost similar flow structure at each level. This is because that these bases have the similar frequency character in six W_j approximation spaces. Figure 4 shows the multiresolution images based on Coifman basis of index N=30. The image of each level is very similar to that of Daubechies basis of index N=18. We also find that the similar multiresolution images also appear in Baylkin basis of index N=18 and Daubechies basis of index N=20. Based on above results, it is reasonable to say that the similar multiresolution images of turbulent structures can be obtained using the wavelet basis with the higher index number, even though wavelet basis is different function. Therefore, it is an important condition that wavelet basis function is anywhere differentiable and more compact in the frequency domain. In our problem, the best choice of orthogonal wavelet basis is Daubechies, Coifman and Baylkin families with index $N \geq 18$.

4. Concluding Remarks

(1) The orthonormal wavelet families (Daubechies, Coifman and Baylkin bases) with index N<10 are inappropriate for multiresolution image analysis of turbulent flow.

(2) The multiresolution images of turbulent structures are very similar when using the wavelet basis with the higher index number, even though wavelet basis is different function.

(3) The condition of the best choice of orthogonal wavelet basis is anywhere differentiable and compact support in the frequency domain

(4) The image components in orthogonal spaces with different scales can be obtained by decomposing the image of turbulent structure using the two-dimensional orthogonal wavelet multiresolution analysis and provide further evident of the multiscale structures in jet.

(5) The edges of the vortices at different resolutions or scales and the coherent structure can be easily extracted from these image components.

References

Brasseur, J. G. and Wang, Q., 1992, "Structural Evolution of Intermittency and Anisotropy at Different Scales Analyzed Using Three-dimensional Wavelet Transforms", *Phys. Fluids A*, Vol.4, No.11, pp.2538-2554.

Catrakis, H. J. and Dimotakis P. E., 1996, "Mixing in Turbulent Jets: Scalar Measures and Isosurface Geometry", *J. Fluid Mech.*, Vol.317, pp.369-406.

Dallard, T. and Browand, F.K., 1993, "Scale Transitions at Defect Sites in the Mixing Layer: Application of the 2-D Arc Wavelet Transform", *J. Fluid Mech.*, Vol.247, pp.339-368.

Dallard, T. and Spedding, G.R., 1993, "2-D Wavelet Transform: Generalisation of the Hardy Space and Application to Experimental Studies", *Eur. J. Mech. B/Fluids*, Vol.12, pp.107-134.

Daubechies, I., 1992, "Ten Lectures on Wavelets", *Society for Industrial and Applied Mathematics*.

Everson, R. and Sirovich, L., 1990, "Wavelet Analysis of the Turbulent Jet", *Phys. Lett.*, Vol.145, No.6, pp.314-322.

Gordeyev, S. V. and Thomas, F. O., 1995, "Measurement of Reynolds Stress Reversal in a Planar Jet by Means of a Wavelet Decomposition", *Turbulent Flows ASME*, FED-Vol.208, pp.49-54.

Gordeyev, S. V., Thomas, F. O. and Chu, H. C., 1995, "Experimental Investigation of Unsteady Jet Shear Layer Dynamics Using a Wavelet Decomposition", *Unsteady Flows ASME*, FED-Vol.216, pp.167-172.

Laufer, J., 1975, "New Trends in Experimental Turbulence Research", *Ann. Rev. Fluid Mech.*, Vol.7, pp.307-320.

Lewalle, J., Petagna, P. and Buresti, G., 1994, "Wavelet Statistics of the Near-Field Structure of a Coaxial Jet", *AIAA Paper* 94-2323.

Li, H., 1997, "Application of Wavelet Cross-Correlation Analysis to a Turbulent Plane Jet", *JSME International Journal, Fluids and Thermal Engineering*, Vol.40, No.1, pp.58-66.

Li, H., 1997, "Wavelet Auto-Correlation Analysis Applied to Eddy Structure Identification of Free Turbulent Shear Flow", *JSME International Journal, Fluids and Thermal Engineering*, Vol.40, No.4, pp.567-576.

Li, H., 1997, "Wavelet Analysis on Coherent Structure Dynamics in a Plane Turbulent Jet", *Experimental Heat Transfer, Fluid Mechanics and Thermodynamics*, Edizioni ETS, PISA, Vol.2, pp.1053-1060.

Li, H., 1997, "Wavelet Reynolds Stress Analysis of Two-Dimentional Vortex Flow", *ASME FEDSM97-3040.*

Li, H., 1997, "Wavelet Velocity Correlation Analysis in a Plane Turbulent Jet", *Proceedings of the 11th Symposium on Turbulent Shear Flows*, Vol.3, pp.3-101~106.

Li, H., 1997, "Turbulent Structure Analysis of a Two-Dimensional Jet Using Wavelets", *Proceedings of the 5th Triennial International Symposium on Fluid Control, Measurement and Visualization*, pp.589-594.

Li, H., 1998, "Identification of Coherent Structure in Turbulent Shear Flow with Wavelet Correlation Analysis", *ASME Journal of Fluids Engineering*, Vol.120, No.4.

Li, H., 1998, "Wavelet Statistical Analysis of the near Field Flow Structure in a Turbulent Jet", *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol.41, No.133.

Li, H. and Nozaki, T., 1995, "Wavelet Analysis for the Plane Turbulent Jet (Analysis of Large Eddy Structure)", *JSME International Journal, Fluids and Thermal Engineering*, Vol.38, No.4, pp.525-531.

Li, H., Nozaki, T., Tabata T. and Oshige S., 1997, "Wavelet Analysis of Flow Structure in a Bounded Jet", *Proceedings of International Conference on Fluid Engineering toward the 21st Century*, Vol.2, pp.589-594.

Li, H., Takei, M., Ochi, M., Saito, Y. and Horii, K., 1998, "Velocity Correlation Analysis in the Near-Field of a Turbulent Jet with Help of Discrete Wavelet Transform", *ASME FEDSM98-482.*

Mallat, S. G., 1998, "A wavelet Tour of Signal Processing", *Academic Press.*

Spedding, G. R., Browand, F. K., Huang N. E. and Long S. R., 1993, "A 2-D Complex Wavelet Analysis of an Unsteady Wind-Generated Surface Wave Field", *Dynamics of Atmospheres and Oceans*, Vol.20, pp.55-77.

Walker, S. H., Gordeyev, S. V. and Thomas, F. O., 1997, "A Wavelet Transform Analysis Applied to Unsteady Aspects of Supersonic Jet Screech Resonance", *Experiments In Fluids*, Vol.22, pp.229-238.

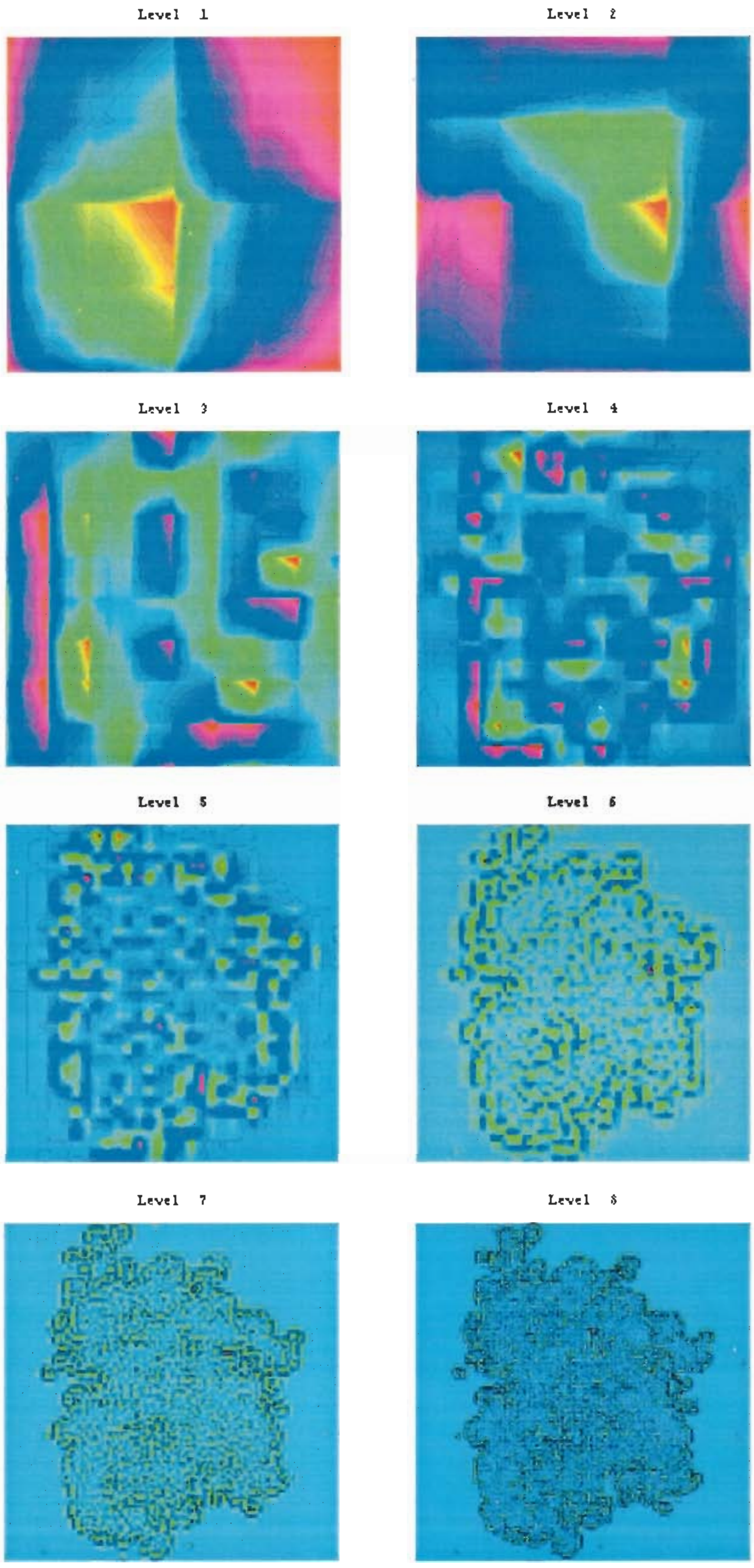

Fig.1 Multiresolution images of a turbulent jet based on Daubechies basis of index *N*=4

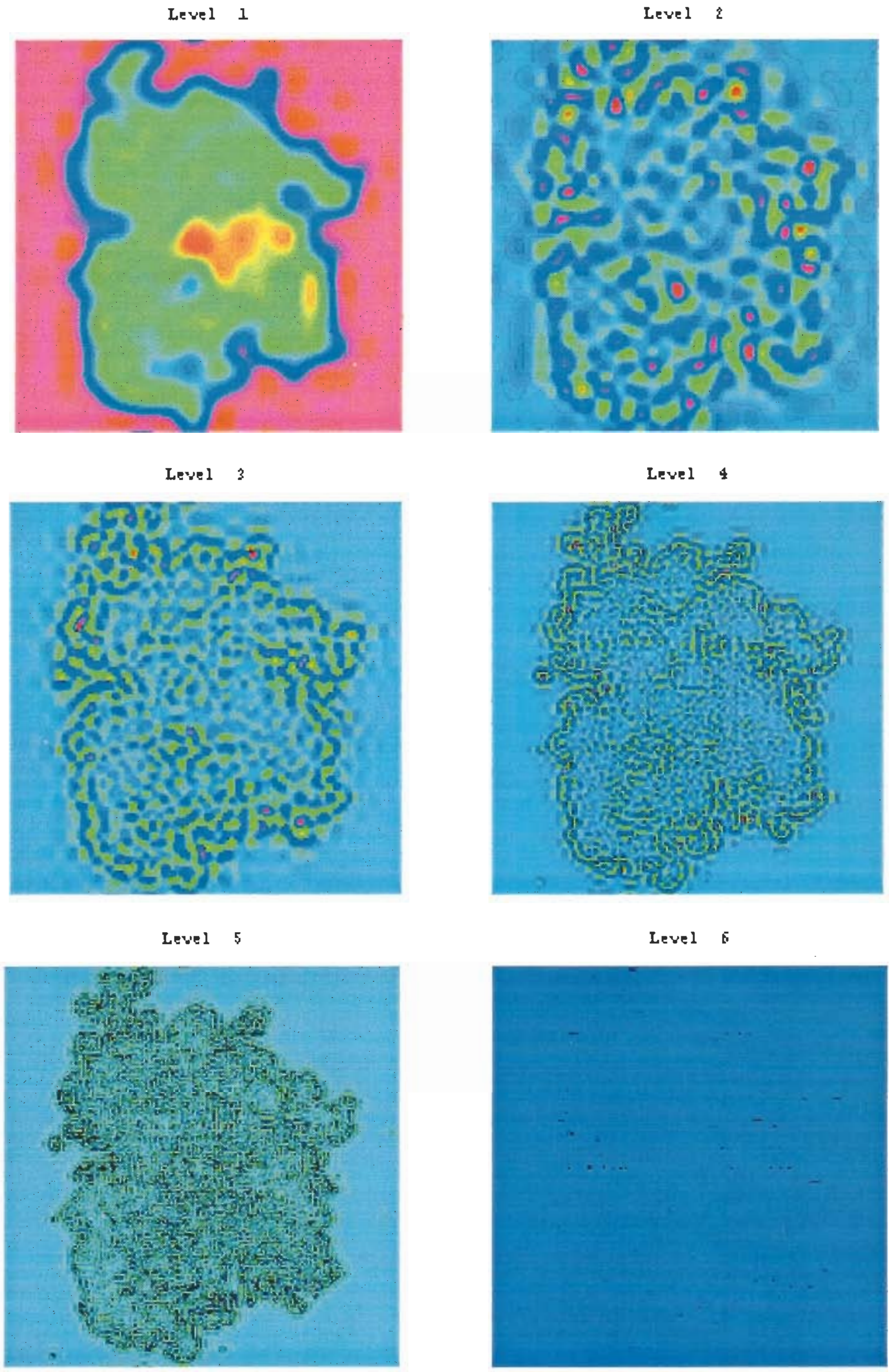

Fig.2 Multiresolution images of a turbulent jet based on Daubechies basis of index N=18

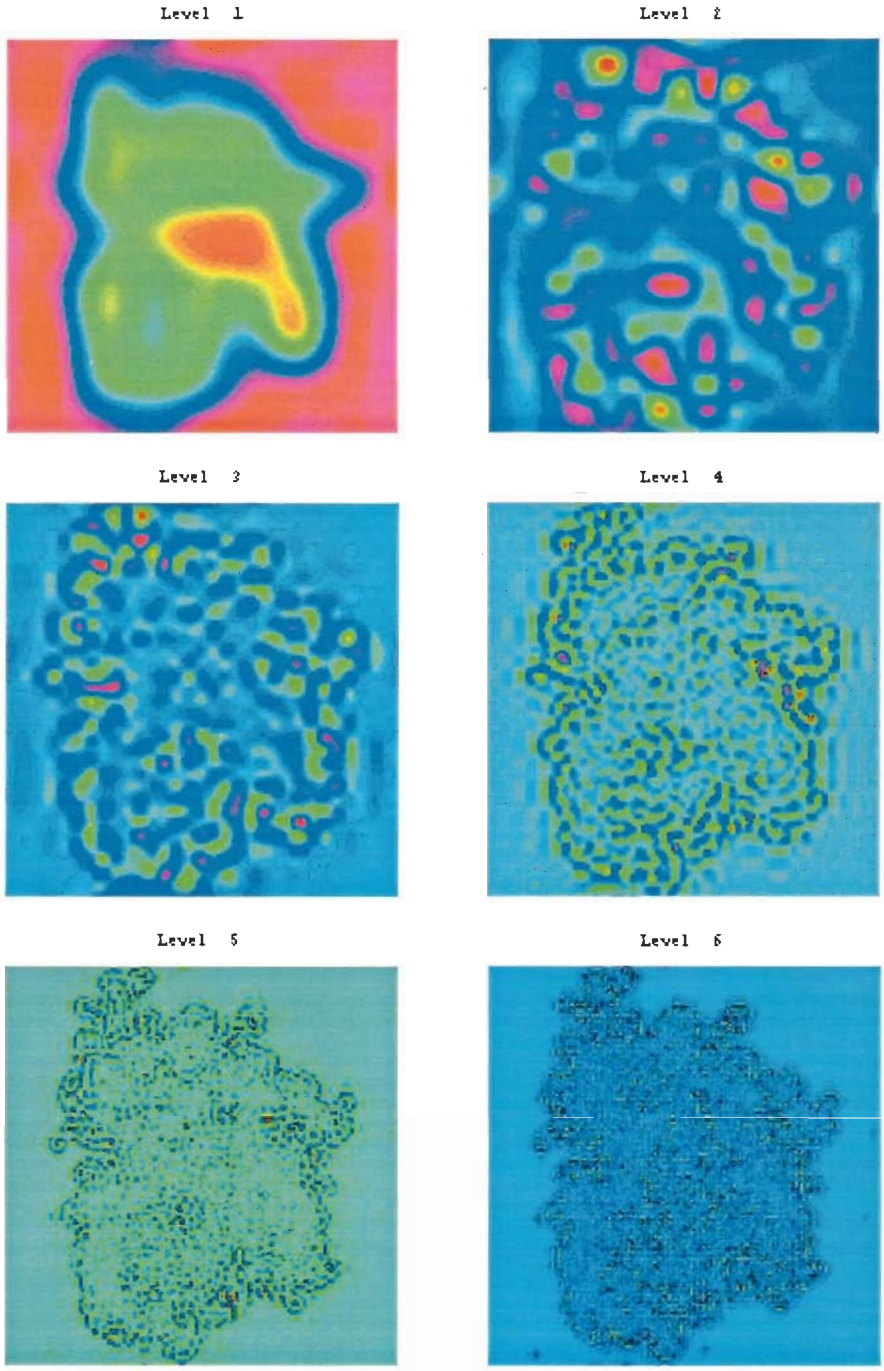

Fig.3 Multiresolution images of a turbulent jet based on Daubechies basis of index *N*=14

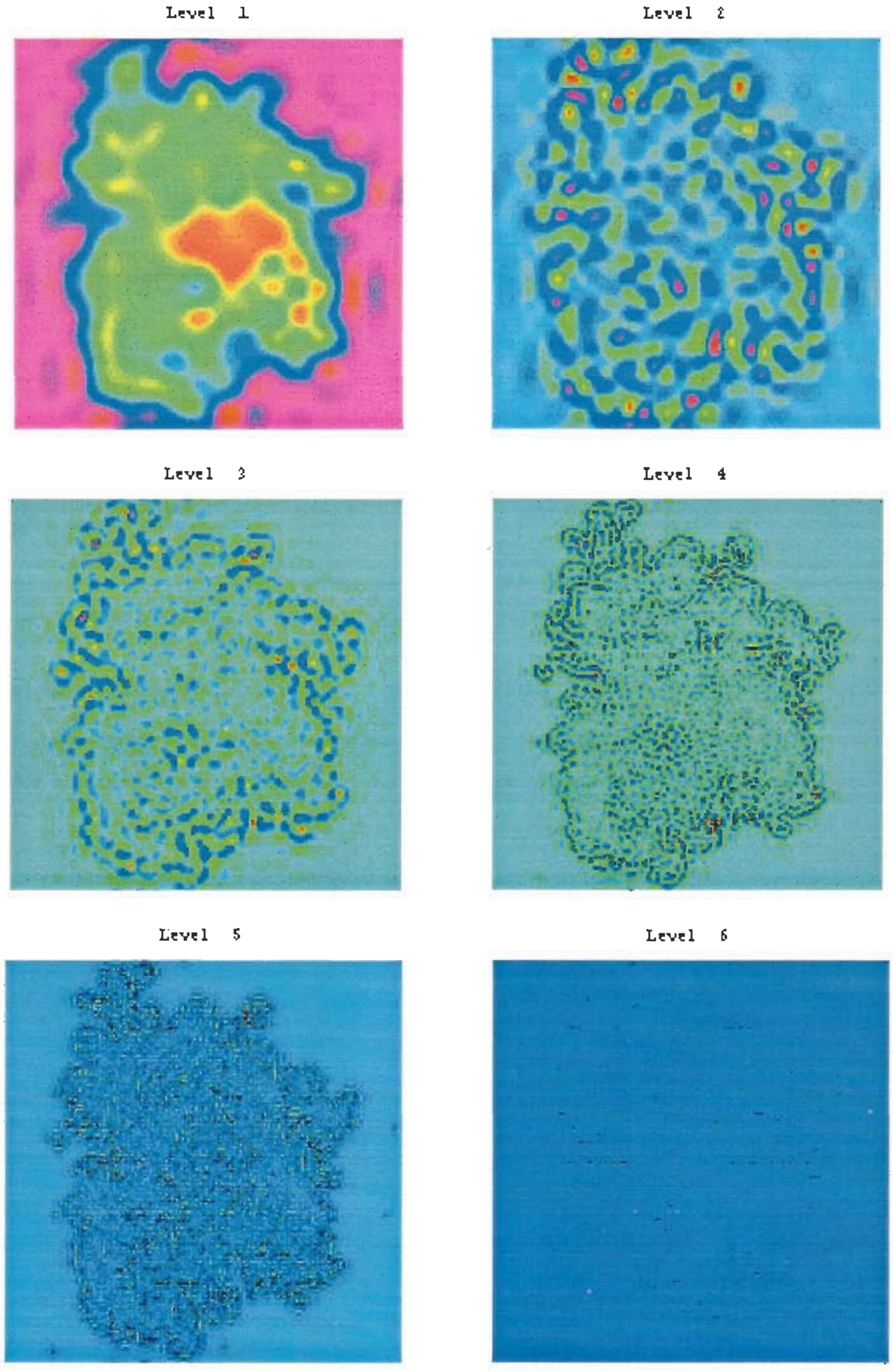

Fig.4 Multiresolution images of a turbulent jet based on Coifman basis of index *N*=30